A Giraud-type Theorem for Model Tòpoi.

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Abstract

Following the unpublished work of [Rzk], we present a formulation of the notion of *model tòpos*, intended as a model-categorical version of the classical concept of *Grothendieck tòpos*. Such a definition is sensible enough to establish a *Giraud-type theorem* for model tòpoi.

We start by reviewing the notion of *Grothendieck tòpos*, albeit from a slightly unusual perspective which avoids the use of *Grothendieck topologies*. We then state one of the possible formulation of the classical *Giraud's theorem* for Grothendieck tòpoi which characterises them axiomatically as categories satisfying suitable internal properties. The concept of *weak descent* is fundamental for this result to hold.

Our definition of Grothendieck tòpoi comprises, up to equivalences of categories, all and only those categories admitting a left exact *small presentation*. Taking this point of view, it is relatively easy to explain how to homotopify the ordinary tòpos-theoretical setting (substituting presheaves categories with simplicial presheaves categories and localizations with Bousfield localizations) and get the desired notion of *model tòpoi*.

We finally state and sketch the proof of a meaningful version of Giraud's theorem for such model tòpoi, implementing the notion of *descent* for model categories. If time permits, we see how this result applies to provide a nice class of examples of model tòpoi which present the homotopy theory of *homotopy sheaves* on a Grothendieck site.

References

[AdRo]   Jiří Adámek, Jiří Rosický, *Locally Presentable and Accessible Categories*, Cambridge University Press, Cambridge, 1994

[Dug1]   Daniel Dugger, *Universal homotopy theories*, Advances in Mathematics, Volume 164, Issue 1, 1 December 2001, Pages 144–176.

[Dug2]   Daniel Dugger, *Combinatorial model categories have presentation*, Advances in Mathematics, Volume 164, Issue 1, 1 December 2001, Pages 177–201

[DHI]   Daniel Dugger, Sharon Hollander, Daniel Isaksen, *Hypercovers and simplicial presheaves*, Mathematical Proceedings of the Cambridge Philosophical Society. Vol. 136. No. 1, 2004.

[Hir]   Philip S. Hirschhorn, *Model Categories and their Localizations*, Mathematical Surveys and Monographs, Volume 99, AMS, 2003.

[McMo]   Saunders Mac Lane, Ieke Moerdijk, *Sheaves in Geometry and Logic. A First Introduction to Topos Theory*, Springer-Verlag, New York, 1992.

[Rzk]   Charles Rezk, *Toposes and Homotopy Toposes*, version 0.15.