

# Who was the mathematician Julius Wolff?

Bas Edixhoven

Universiteit Leiden

Cleveringa lecture  
Nederlandse Ambassade

Julius Wolff (born in Nijmegen, April 18, 1882, deceased in Bergen-Belsen, February 8, 1945), was a Dutch mathematician.

After obtaining his PhD in Amsterdam, he first taught in secondary schools in Meppel, Middelburg and Amsterdam before he became a professor at the universities of Groningen and Utrecht.

He was a very productive advisor of PhD students. Between 1918 and 1940, he was the advisor of 25 students. Among them was Cornelis Visser, who later became professor of applied mathematics in Leiden.

In my lecture I will discuss, without technicalities, the work of Julius Wolff and his importance for mathematics in the Netherlands.

# Background of the Cleveringa lectures

On Saturday November 23 of 1940, the jewish personnel of the dutch universities was removed from their positions by the ministry of education.



On Tuesday November 26, Rudolph Cleveringa, then dean of the faculty of law, held his famous protest speech.

He gave his speech at the time and place of the class of Eduard Maurits Meijers, one of the removed professors.

Also Ton Barge (anatomy and embryology) and Lambertus van Holk (theology) protested publicly.

The students went on strike, and the university was closed by the german authorities.

# The university of Leiden during the occupation

The closure of the university was fortunate, because the German authorities had plans to reorganise the university according to Nazi ideology.

These plans led to a continuous struggle between the German authorities and the board and professors of the university.

After some professors were fired in March and April of 1942, 53 professors and 3 lecturers collectively resigned from their positions. The university and the German plans came to a full stop.

Sources:

Mr. P.J. Idenburg, "De Leidse Universiteit 1928–1946", Universitaire Pers, Leiden, 1978 (ISBN 90.6021.425.0);

Prof. dr. W. van der Woude's lecture in "Kort Verhaal van de plechtige heropening der universiteit. . .", Universitaire Pers Leiden, 19??.

Anecdote!

# Why do I give a Cleveringa lecture?

Cleveringa was arrested and imprisoned in Scheveningen, but survived the war. Also Meijers, Barge and van Holk survived the war.

Telders did not survive the war, he died in Bergen-Belsen in April 1945. The “Teldersstichting” is named after him.

I want to speak about Julius Wolff, a jewish dutch professor of mathematics in Utrecht, who was in a similar position as Meijers, but did not survive the war.

My goal is that more people remember Wolff.

A bit after I came to Leiden, as professor, in 2002, I stumbled on Wolff's inaugural lecture: “De nieuwere onderzoekingen op het gebied der algebraïsche oppervlakken”, Amsterdam, 1916.

I was surprised that I had never heard of him, as algebraic geometry is my trade.

# Short CV of Julius Wolff



Prof. Dr. J. WOLFF †

Oyano Oct. 1931

Born: 18 April 1882, Nijmegen, son of Levie Wolff, and Ida Jacobsohn.

Married Betsy Gersons, 9 August 1911 in Tilburg. Three children.

Studied maths and phys, UvA, PhD 1908. Teacher in Meppel, Middelburg and Amsterdam, 1907–1917.

Professor RUG (1917), UU (1922).

Concentration camps: Barneveld, Westerbork, Bergen-Belsen.

Died: 8 February 1945, Bergen-Belsen.

Sources: wikipedia (Julius Wolff mathematician), and <http://www.joodsmonument.nl/page/446318/en>, and <http://dbnl.org/index.php>, Geestelijk Nederland, 1920-1940.

# Mathematical production

Publications: about 145 articles (some seem to be counted twice), 4 books.

PhD students: 25.

Willem Burgers UU 1929

Gerrit Deinema RUG 1918

Frans de Kok UU 1932

Bastiaan Grootenboer UU 1932

Herman Looman UU 1923

Johannes Nagel UU 1929

Joël Rozenberg UU 1925

Jan van de Putte UU 1927

Jan van Kuik UU 1940

Cornelis Visser UU 1935

Pieter Vredenduin UU 1931

Berend Wever UU 1931

Egbertha Zwanenburg RUG 1918

Cornelis Campagne UU 1929

Jan Deknatel UU 1935

Adrianus Dubbeld UU 1932

Johannes Hoekstra UU 1927

Johanna Marx UU 1930

Frederik Nijhoff UU 1927

Johannes Thie UU 1924

Albertus van Haselen UU 1929

Mels van Vlaardingen UU 1936

Sigofred Vles UU 1939

Johan Wansink UU 1931

Wilhelm Wieringa RUG 1918

# Wolff's mathematical contributions are alive

Wolff is well known for his work on functions of a complex variable, in particular for the Denjoy-Wolff theorem.

He had humor, even in the choice of the titles of his articles:

Sur une généralisation d'un théorème de Schwarz, CRAS, 1926.

Sur une généralisation d'un théorème de Schwartz, CRAS, 1926.

He wrote in dutch, french, german, and english.

He was praised for his efficient and clear exposition.

He was one of the more important dutch mathematicians in his time.

For example, he is mentioned (pages 97, 98 and 100) of “De ontwikkeling van de natuurwetenschappen. . .” produced for the international exposition in Luik, 1930.

Let us look at a recent article:

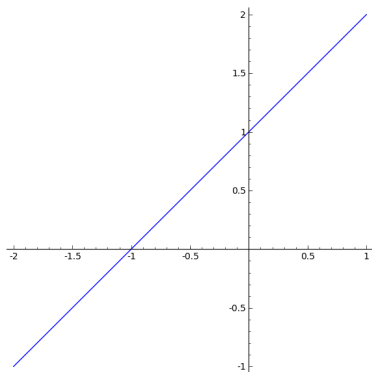
Harold P. Boas, “Julius and Julia: mastering the art of the Schwarz lemma.” Amer. Math. Monthly 117 (2010), no. 9, 770–785.



# Algebraic geometry, from Wolff to today

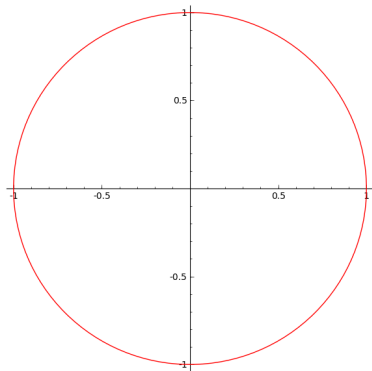
Wolff described the state of the art in his inaugural lecture (oratie), in 1916.

Back to René Descartes (1630).



The line given by the equation  $y = x + 1$ .

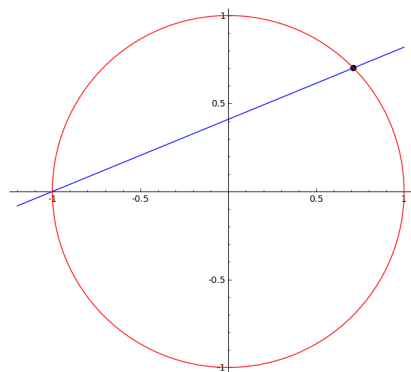
# A plane curve of degree 2



The circle given by the equation  $x^2 + y^2 = 1$ .

# Algebraic parametrisation of the circle

Let us look at page 4 of Wolff's inaugural lecture.



Lines through  $(-1, 0)$ :

$$y = a \cdot (x + 1).$$

Second point of intersection:

$$\left( \frac{1 - a^2}{1 + a^2}, \frac{2a}{1 + a^2} \right)$$

Algebraic parametrisation:  $\mathbb{R} \rightarrow \text{circle}$ ,  $a \mapsto \left( \frac{1 - a^2}{1 + a^2}, \frac{2a}{1 + a^2} \right)$ .

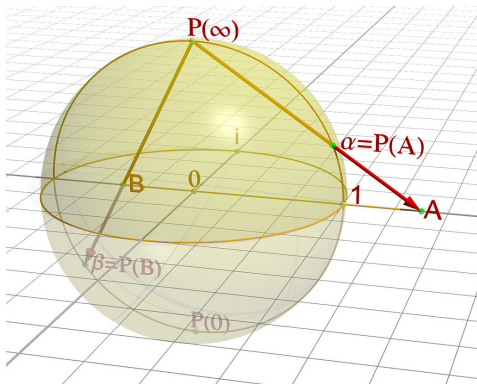
Analytic parametrisation:  $\mathbb{R} \rightarrow \text{circle}$ ,  $x \mapsto (\cos(x), \sin(x))$ .

# Analytic and geometric approaches

We look at page 5 (bottom). Wolff mentions two approaches: complex analytic and geometric (also called algebraic because one uses only algebraic functions).

The complex analytic approach uses complex numbers:  $a + bi$  with  $a$  and  $b$  real,  $i^2 = -1$ .

The set of complex points of a line is a plane, and can also be seen as a sphere minus a point:



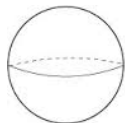
# Genus of a plane curve: complex analytic approach

Our algebraic parametrisation of the circle, but now complex:

$$\mathbb{C} - \{i, -i\} \rightarrow \text{complex circle}, \quad a \mapsto \left( \frac{1 - a^2}{1 + a^2}, \frac{2a}{1 + a^2} \right)$$

shows that the complex circle minus the point  $(-1, 0)$  is the complex plane minus two points (the solutions of  $a^2 + 1 = 0$  are  $i$  and  $-i$ ).

So, the complex analytic approach shows that the complex circle and the complex line both differ from the sphere by 2 or 1 points. They are called of genus zero because of this.



genus 0



genus 1



genus 2

Higher genus:

# Genus of a curve: algebraic/geometric approach

We are now on page 6 of Wolff's inaugural lecture.

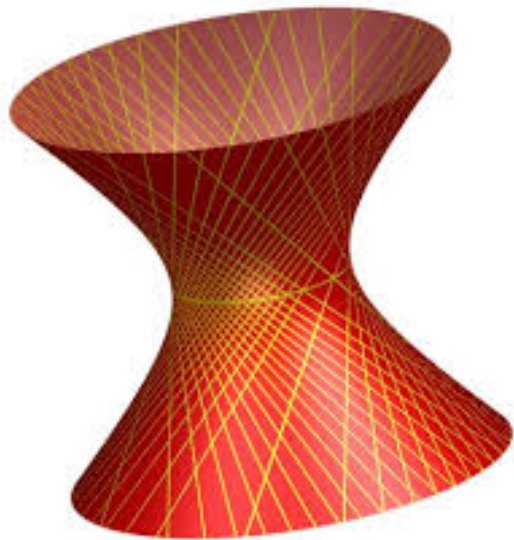
The genus of a curve can be seen algebraically in terms of the *dimension* of the set of algebraic functions with prescribed poles. But to understand what this means, one has to study mathematics for say a year. Nevertheless, it is this interplay between the two approaches that is extremely important in mathematics.

In the rest of Wolff's lecture, one sees that in 1916 there were still many problems in understanding the generalisation to surfaces (one dimension higher).

The tools for solving these problems were developed during and directly after the war, starting with Jean Leray (as prisoner of war).

Nowadays, a student in mathematics in the Netherlands can learn these tools from the third year on.

# A surface of degree 2

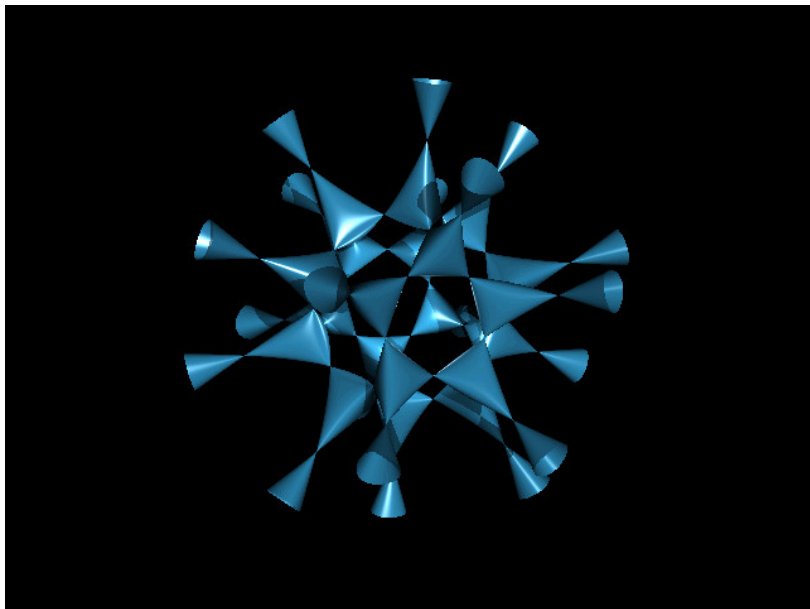


# A surface of degree 3 (Clebsch diagonal surface)





# A surface of degree 6 (Barth sextic)



## Some links for the pictures, 3d-prints

Sage server Leiden: <https://sage.math.leidenuniv.nl/>  
Create an account, search for the worksheet  
“MSTcalculus2” by Bas Edixhoven.

For 3d-printing: <http://blog.mo-labs.com/2012/08/19/the-clebsch-diagonal-surface/>

Check out also: <http://blog.mo-labs.com/laser-in-glass/>

For mathematical art:

<http://www.shapeways.com/art/mathematical-art?li=nav>

For jewelry etc.: <http://www.shapeways.com/jewelry?li=nav>

Thank you for your attention!

Let us watch

<https://www.youtube.com/watch?v=3ryBzWNhLxY>