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EXTRA HERKANSING CONTINUE WISKUNDE 1, 24/1/2022

1 a) Eventuele geheeltallige nulpunten van $f(x) = x^3 - 3x^2 + 4$ zijn delers van 4, dus $\pm 1, \pm 2, \pm 4$.
Er geldt: $f(1) = 1 - 3 + 4 = 2$, $f(-1) = -1 - 3 + 4 = 0$. Dus $x = -1$ is een nulpunt. We krijgen de andere ~~delers van~~ nulpunten van f door $f(x)$ te delen door $x - (-1) = x + 1$:

$$
\begin{array}{l}
x+1 \,/\, x^3 - 3x^2 + 4 \,\backslash\, x^2 - 4x + 4 \\
\underline{x^3 + x^2} \quad - \\
\quad -4x^2 + 4 \\
\quad \underline{-4x^2 - 4x} \\
\qquad 4x + 4 \\
\qquad \underline{4x + 4} \\
\qquad\quad 0
\end{array}
$$

$$
\begin{aligned}
x^3 - 3x^2 + 4 &= (x+1)(x^2 - 4x + 4) \\
&= (x+1)(x-2)^2
\end{aligned}
$$

Dus de nulpunten van $f(x)$ zijn $x = -1$, $x = 2$

b) $f'(x) = 3x^2 - 6x = 3(x^2 - 2x) = 3x(x-2)$

Tekenoverzicht van f'

$$
\begin{array}{ccccccccc}
 & + & 0 & - & 0 & + & \\
[-4 & & 0 & & 2 & & 4] \\
 & x<0 & & x>0 & & x>0 & \\
 & x-2<0 & & x-2<0 & & x-2>0 & \\
f & \nearrow & & \searrow & & \nearrow &
\end{array}
$$

f stijgt tussen -4 en 0, daalt tussen 0 en 2, en stijgt tussen 2 en 4

x	$f(x)$ grootte	aard
-4	$-64 - 48 + 4 = -108$	absolut minimum
0	4	relatief maximum
2	0	relatief minimum
4	$64 - 48 + 4 = 20$	absolut maximum

(2)

① c) $x^2+8 \,/\, x^3-3x^2+4 \,|\, x-3$

$$\begin{array}{l} x^3 \quad +8x \\ \hline -3x^2-8x+4 \\ -3x^2 \quad -24 \\ \hline -8x+20 \end{array}$$

$x^3-3x^2+4 = (x-3)(x^2+8) + (-8x+20)$

$g(x) = x-3 + \dfrac{-8x+20}{x^2+8}$

$\lim\limits_{x\to\pm\infty} g(x)-(x-3) = \lim\limits_{x\to\pm\infty} \dfrac{-8x+20}{x^2+8}$

$= \lim\limits_{x\to\pm\infty} \dfrac{-8/x+20/x^2}{1+8/x^2} = 0$

Dus $\boxed{y = x-3}$ is de schieve asymptoot van $g(x)$ voor $x\to\pm\infty$

② $y = \dfrac{4}{x^2}$ $\quad x^2+y^4 = x^2 + \dfrac{4^4}{x^8} = x^2 + \dfrac{256}{x^8} = f(x)$

$f'(x) = 2x - \dfrac{8\times 256}{x^9} = \dfrac{2x^{10}-8\times256}{x^9} = \dfrac{2(x^{10}-1024)}{x^9}$

tekenoverzicht f': $\quad \begin{array}{c|ccc} & - & 0 & + \\ \hline 0 & \searrow & 2 & \nearrow \end{array}$ $\quad 2^{10} = 1024$

f neemt zijn absolute minimum aan in $\boxed{x=2}$

De bijbehorende y-waarde is $\boxed{y = \dfrac{4}{2^2} = 1}$

③ $\lim\limits_{x\uparrow 1} f_c(x) = \lim\limits_{x\uparrow 1} \dfrac{4}{cx} = \dfrac{4}{c}$

$f_c(x)$ linkscontinu in $x=1 \Leftrightarrow \lim\limits_{x\uparrow 1} f_c(x) = f_c(1) \Leftrightarrow \dfrac{4}{c} = c \Leftrightarrow c^2 = 4$

$\Leftrightarrow \boxed{c = 2 \text{ of } c = -2}$

$\lim\limits_{x\downarrow 1} f_c(x) = \lim\limits_{x\downarrow 1} 6\cos\pi x - c^2 - 4c = -6-c^2-4c$

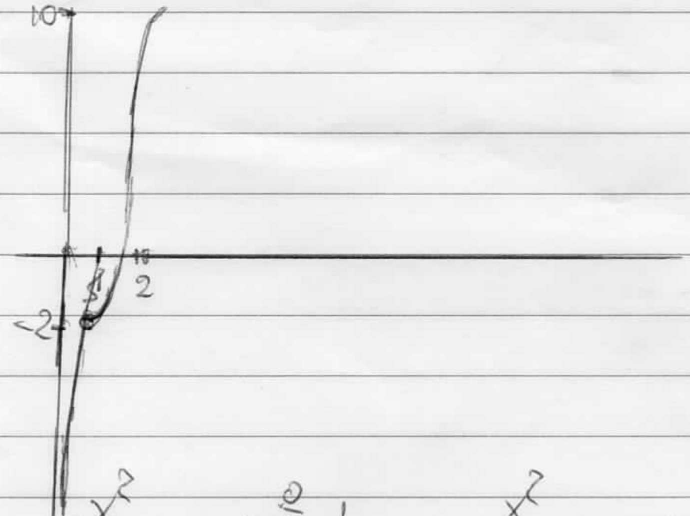
$f_c(x)$ rechtscontinu in $x=1 \Leftrightarrow \lim\limits_{x\downarrow 1} f_c(x) = f_c(1) \Leftrightarrow -6-c^2-4c = c$

$\Leftrightarrow c^2+5c+6 = 0 \Leftrightarrow (c+2)(c+3) = 0 \Leftrightarrow \boxed{c=-2 \text{ of } c=-3}$

$f_c(x)$ continu in $x=1 \Leftrightarrow f_c(x)$ linkscontinu en rechtscontinu in $x=1$

$\Leftrightarrow \boxed{c = -2}$

b) $f_2(x) = \begin{cases} -2/x & \text{voor } 0<x<1 \\ -2 & \text{voor } x=1 \\ 6\cos \pi x + 4 & \text{voor } x>1 \end{cases}$



4 a) $\lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2} \overset{0/0}{=} \lim_{x\to 0} \dfrac{e^{x^2}.2x + \sin x}{2x}$

$\overset{\text{l'Ho}}{=} \lim_{x\to 0} \dfrac{e^{x^2}.2 + e^{x^2}(2x).2x + \cos x}{2} = \dfrac{2+1}{2} = \boxed{\dfrac{3}{2}}$

b) Schrijf $x^{1/x^2} = e^{\ln x / x^2}$

$\lim_{x\to\infty} \dfrac{\ln x}{x^2} = 0$ (standaard limiet). Dus $\lim_{x\to\infty} x^{1/x^2} = e^0 = \boxed{1}$

5 a) $f(x) = \dfrac{1}{8x^9 - 9x^8}$ $\quad 8x^9 - 9x^8 = 0 \Leftrightarrow x^8(8x-9) = 0 \Leftrightarrow x=0$ of $x = \frac{9}{8}$

Domein $\mathbb{R}\setminus\{0, \frac{9}{8}\}$

Tekenoverzicht f:

$$\begin{array}{ccccc} - & NG & - & NG & + \\ & 0 & & 9/8 & \end{array}$$

$x^8 \geq 0$, $8x-9<0$, $8x-9>0$

verticale asymptoten: $x=0$, $x=\frac{9}{8}$

$\lim_{x\downarrow \frac{9}{8}} f(x) = \infty$, $\lim_{x\uparrow \frac{9}{8}} f(x) = -\infty$, $\lim_{x\downarrow 0} f(x) = -\infty$, $\lim_{x\uparrow 0} f(x) = -\infty$

b) graad teller < graad noemer, dus er is een horizontale asymptoot ~~y=0~~ voor $x\to\pm\infty$

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \dfrac{1}{8x^9 - 9x^8} = \lim_{x\to\pm\infty} \dfrac{x^{-9}}{8 - 9x^{-1}} = 0$

④

5c) $f'(x) = \left(\left(8x^9-9x^8\right)^{-1}\right)' = (-1)\left(8x^9-9x^8\right)^{-2}\cdot\left(8x^9-9x^8\right)'$

$= (-1)\dfrac{72x^8-72x^7}{\left(8x^9-9x^8\right)^2} = \dfrac{(-72)\,x^7(x-1)}{(x^8)^2(8x-9)^2} = \dfrac{(-72)(x-1)}{x^9(8x-9)^2}$

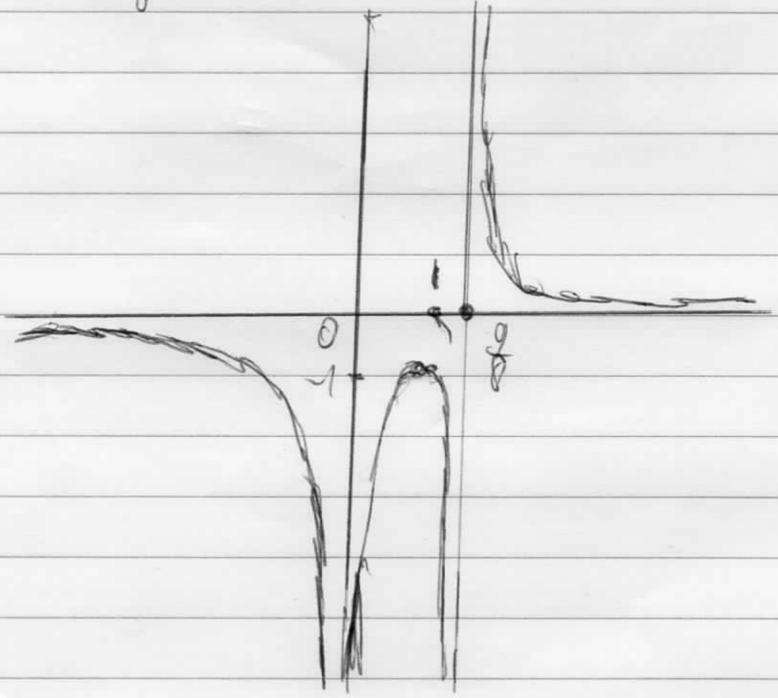
d) tekenoverzicht f'

	$-$	NG	$+$	0	$-$	NG	$-$
		0		1		$\frac{9}{8}$	
	$x<0$, $x<1$		$x>0$, $x<1$		$x>0$, $x>1$		$x>0$, $x>1$
f	↘		↗		↘		↘

extremen $x=1$ $f(1) = \frac{1}{8-9} = -1$ <u>relatief</u> maximum

relatief, want $\lim_{x\downarrow\frac{9}{8}} f(x) = \infty$

e)



⑥ $f(x) = \sqrt[4]{1+10x} = (1+10x)^{1/4}$

$f'(x) = \frac{1}{4}(1+10x)^{-3/4}\cdot 10 = \frac{5}{2}(1+10x)^{-3/4}$

$f^{(2)}(x) = \frac{5}{2}\cdot -\frac{3}{4}(1+10x)^{-7/4}\cdot 10 = -\frac{75}{4}(1+10x)^{-7/4}$

$f^{(3)}(x) = -\frac{75}{4}\cdot\left(-\frac{7}{4}\right)\cdot(1+10x)^{-11/4}\cdot 10 = \frac{2625}{8}(1+10x)^{-11/4}$

(6) b) $P_{2,8}(x) = f(8) + f'(8)(x-8) + \frac{f^{(2)}(8)}{2!}(x-8)^2$

$f(8) = (1+8\cdot 10)^{1/4} = 81^{1/4} = 3$

$f'(8) = \frac{5}{2}\, 81^{-3/4} = \frac{5}{2}\times 3^{-3} = \frac{5}{54}$

$f^{(2)}(8) = -\frac{75}{4}\times 3^{-7} = \frac{-25}{4}\times 3^{-6} = \frac{-25}{4\times 729} = \frac{-25}{2916}$

$f^{(2)}(8)/2! = \frac{-25}{5832}$ $\boxed{P_{2,8}(x) = 3 + \frac{5}{54}(x-8) - \frac{25}{2916}(x-8)^2}$

c) $R_{3,8}(x) = \frac{f^{(3)}(s)}{3!}(x-8)^3$ s between 8 and x

$= \frac{2625/8}{6}(1+10s)^{-11/4}\cdot(x-8)^3 = \boxed{\frac{875}{16}(1+10s)^{-11/4}(x-8)^3}$