UITWERKING HERKANSING CONTINUE WISKUNDE 1 21/12/2021

1 a) Eventuele geheeltallige nulpunten van f zijn positieve of negatieve delers van 2, dus ±1, ±2.
Er geldt $f(1) = 1-6+9-2 = 2$, $f(-1) = -1-6-9-2 = -18$, $f(2) = 8-24+18-2 = 0$
Dus $x=2$ is een nulpunt

$$x-2 \,/\, x^3-6x^2+9x-2 \,\backslash\, x^2-4x+1 \qquad x^3-6x^2+9x-2 = (x-2)(x^2-4x+1)$$

$$\begin{array}{l} \underline{x^3-2x^2} \; - \\ -4x^2+9x-2 \\ \underline{-4x^2+8x} \; - \\ x-2 \\ \underline{x-2} \\ 0 \end{array}$$

Nulpunten van x^2-4x+1:

$$\frac{4 \pm \sqrt{4^2-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \tfrac{1}{2}\sqrt{12} = 2 \pm \sqrt{3}$$

$(\sqrt{12} = \sqrt{2^2 \cdot 3} = 2\sqrt{3})$

Dus de nulpunten van $f(x)$ zijn $\boxed{2,\ 2 \pm \sqrt{3}}$

b) $f'(x) = 3x^2 - 12x + 9 = 3(x^2-4x+3) = 3(x-1)(x-3)$

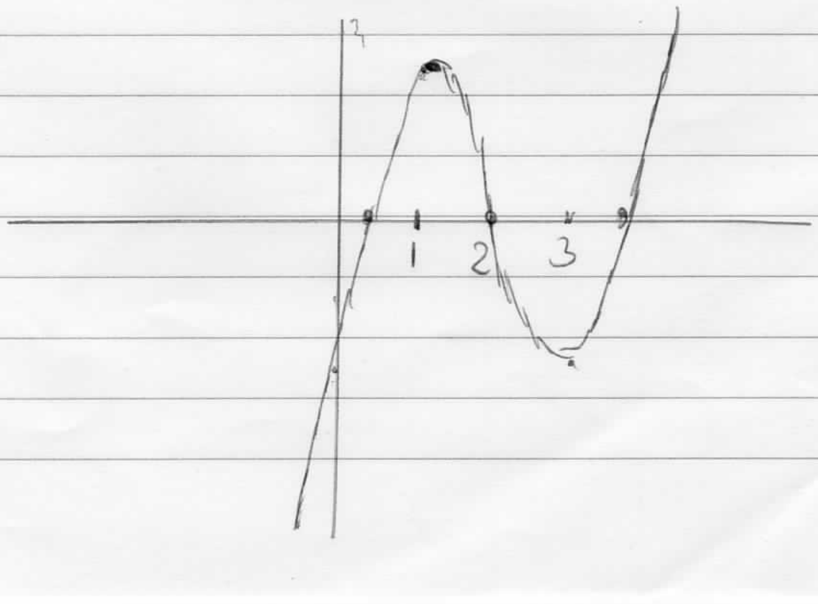
Tekenoverzicht f': $+$ (1) $-$ (3) $+$

f: $\nearrow$ $\searrow$ $\nearrow$

extremen: plaats grootte aard

1 $f(1) = 2$ relatief max want $\lim_{x \to \infty} f(x) = \infty$

3 $f(3) = -2$ relatief min want $\lim_{x \to -\infty} f(x) = -\infty$



① c) $x^2+2 \,/\, x^3-6x^2+9x-2 \,\backslash\, x-6$

$$\begin{array}{l} x^3+2x \quad - \\ \hline -6x^2+7x-2 \\ -6x^2-12 \quad - \\ \hline 7x+10 \end{array}$$

x^3-6x^2+9x-2

$= (x-6)(x^2+2)+7x+10$

$\frac{x^3-6x^2+9x-2}{x^2+2} = x-6+\frac{7x+10}{x^2+2}$

$$\lim_{x\to\pm\infty} [g(x)-(x-6)] = \lim_{x\to\pm\infty} \frac{7x+10}{x^2+2} = \lim_{x\to\pm\infty} \frac{\frac{7}{x}+\frac{10}{x^2}}{1+\frac{2}{x^2}} = 0$$

Dus $\boxed{y=x-6}$ is een schuine asymptoot van $g(x)$ naar $x\to\pm\infty$

② Gegeven $x\geq 0, y\geq 0, \sqrt{x^2+10y^2}=1$ dus $x^2+10y^2=1$, $y^2=\frac{1-x^2}{10}$

Inhoud blok $3xy^2 = 3x\cdot\frac{1-x^2}{10} = \frac{3}{10}x(1-x^2) = \frac{3}{10}x-\frac{3}{10}x^3 = f(x)$

We moeten het absolute maximum bepalen van $f(x)$ op $[0,\infty)$
Er geldt $f'(x) = \frac{3}{10}-\frac{9}{10}x^2$ dus $f'(x)=0$ als $x^2=\frac{3/10}{9/10}=\frac{1}{3}$,
$x=\sqrt{\frac{1}{3}}=\frac{1}{3}\sqrt{3}$ (we hoeven niet naar $x<0$ te kijken)

Tekenverloop f'

x	0		$\sqrt{\frac{1}{3}}$	
f'		$+$		$-$
f		$\nearrow$		$\searrow$

Dus f neemt op $[0,\infty)$ een absoluut maximum aan in $\boxed{x=\sqrt{\frac{1}{3}}}$
De bijbehorende y-waarde is

$$\sqrt{\frac{1-x^2}{10}} = \sqrt{\frac{1-\frac{1}{3}}{10}} = \sqrt{\frac{2/3}{10}} = \boxed{\sqrt{\frac{1}{15}}}$$

3 a) $\lim_{x \uparrow 0} f_c(x) = \lim_{x\uparrow 0} 4\cdot c^{x-1} = 4\cdot c^{0-1} = \frac{4}{c}$

$f_c(0) = 5-c$

f_c linkscontinu in $x=0$ $\Leftrightarrow \lim_{x\uparrow 0} f_c(x) = f_c(0) \Leftrightarrow \frac{4}{c} = 5-c \Leftrightarrow 4=(5-c)c = 5c-c^2$

$\Leftrightarrow c^2-5c+4=0 \Leftrightarrow (c-1)(c-4)=0 \Leftrightarrow \boxed{c=1 \text{ of } c=4}$

f_c rechtscontinu in $x=0$ $\Leftrightarrow \lim_{x\downarrow 0} f_c(x)=f_c(0) \Leftrightarrow \lim_{x\downarrow 0} c^{1-x}-3 = 5-c$

$\Leftrightarrow c-3=5-c \Leftrightarrow 2c=8 \Leftrightarrow \boxed{c=4}$

f_c continu in $x=0$ $\Leftrightarrow$ f_c linkscontinu in $x=0$ en rechtscontinu in $x=0$ $\Leftrightarrow \boxed{c=4}$

b) $\lim_{x\to\infty} a^x = \begin{cases} \infty & \text{als } a>1 \\ 1 & \text{als } a=1 \\ 0 & \text{als } 0<a<1 \end{cases}$

Dus $\lim_{x\to\infty} f_c(x) = \lim_{x\to\infty} c^{1-x}-3 = \begin{cases} \infty & \text{als } 0<c<1 \\ -2 & \text{als } c=1 \\ -3 & \text{als } c>1 \end{cases}$

4 a) $\lim_{x\to 1} \frac{\sin\pi x + \pi\cdot\ln x}{(x-1)^2} \overset{0/0}{=} \lim_{x\to 1} \frac{\pi\cos\pi x + \frac{\pi}{x}}{2(x-1)} \overset{0/0}{=}$

$= \lim_{x\to 1} \frac{-\pi^2\sin\pi x - \frac{\pi}{x^2}}{2} = \boxed{\frac{-\pi}{2}}$

b) $\lim_{x\to\infty} \frac{3^x+2^x+x^{100}}{3^x-x^{80}} = \lim_{x\to\infty} \frac{1+\frac{2^x}{3^x}+\frac{x^{100}}{3^x}}{1-\frac{x^{80}}{3^x}}$

OK

$= \lim_{x\to\infty} \frac{1+\underbrace{\left(\frac{2}{3}\right)^x}_{\to 0} + \underbrace{\frac{x^{100}}{3^x}}_{\to 0}}{1-\underbrace{\frac{x^{80}}{3^x}}_{\to 0}} = \boxed{1}$

④

⑤ $f(x) = \dfrac{\frac{1}{4}x^4+4}{x^4-1}$

a) Domein $x^4 \neq 1$, $x \neq -1$. Dus $\mathbb{R} \setminus \{-1, 1\}$

Tekenoverzicht f: $\frac{1}{4}x^4+4$ is altijd >0

$$\begin{array}{ccccccc} + & NG & - & NG & + \\ \hline & -1 & & 1 & \end{array}$$

$x^4-1>0$ $\quad x^4-1<0$ $\quad x^4-1>0$

Verticale asymptoten: $x=1$, $x=-1$

$\lim_{x \downarrow 1} f(x) = \infty$, $\lim_{x \uparrow 1} f(x) = -\infty$, $\lim_{x \downarrow -1} f(x) = -\infty$, $\lim_{x \uparrow -1} f(x) = \infty$

b) graad teller = graad noemer, dus er is een horizontale asymptoot voor $x \to \pm\infty$

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \dfrac{\frac{1}{4}+4x^{-4}}{1-x^{-4}} = \boxed{\frac{1}{4}}$ $\quad \boxed{y=\frac{1}{4}}$ horizontale asymptoot voor $x \to \pm\infty$

c) $f'(x) = \dfrac{(x^4-1)\cdot x^3 - (\frac{1}{4}x^4+4)\cdot 4x^3}{(x^4-1)^2} = \dfrac{x^7 - x^3 - x^7 - 16x^3}{(x^4-1)^2} = \dfrac{-17x^3}{(x^4-1)^2}$

d) Tekenoverzicht f':

$$\begin{array}{ccccccc} + & & + & & - & & - \\ \hline & -1 & & 0 & & 1 & \\ \nearrow & & \nearrow & & \downarrow & & \downarrow \end{array}$$

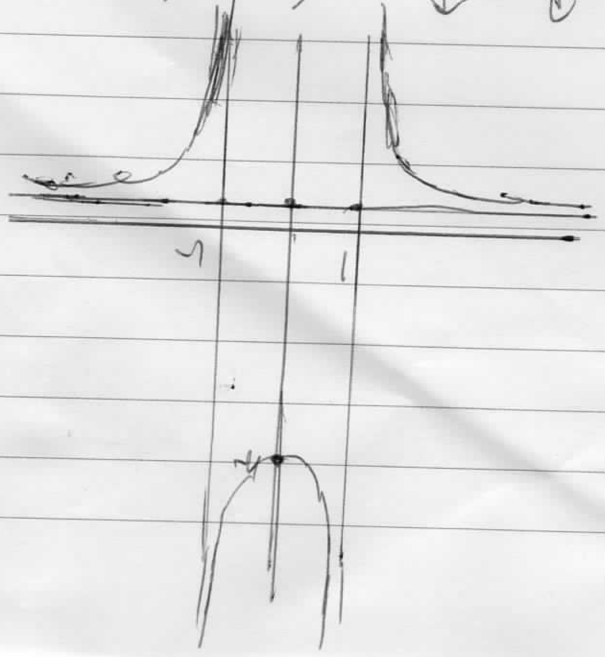
f

$x=0$ relatief max

$f(0) = \frac{4}{-1} = -4$

$\lim_{x \downarrow 1} f(x) = \infty$

e)



6 a) $f(x) = e^{e^x - 1}$

$f'(x) = e^{e^x - 1} \cdot e^x = e^{e^x + x - 1}$

$f''(x) = e^{e^x + x - 1} \cdot (e^x + x - 1)' = e^{e^x + x - 1} \cdot (e^x + 1) = e^{e^x + 2x - 1} + e^{e^x + x - 1}$

$f'''(x) = e^{e^x + 2x - 1} \cdot (e^x + 2) + e^{e^x + x - 1} \cdot (e^x + 1)$

b) $P_{2,0}(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$

$f(0) = e^{e^0 - 1} = e^0 = 1,$ $f''(0) = e^{e^0 + 0 - 1} + e^{e^0 + 0 - 1} = 2$

$f'(0) = e^{e^0 + 0 - 1} = 1$

$P_{2,0}(x) = 1 + x + \frac{2}{2!}x^2 = \boxed{1 + x + x^2}$

c) $R_{3,0}(x) = \frac{f'''(s)}{3!}x^3$ s mellom 0 og x

$= \frac{e^{e^s + 2s - 1}(e^s + 2) + e^{e^s + s - 1}(e^s + 1)}{6}x^3$