Herkansing Continue Wiskunde deel 2
10 maart 2014.

1 a) $\int_0^\infty \frac{x}{(x^2+1)^2}dx$. Eerst primitiveren bepalen

$$\int \frac{x\,dx}{(x^2+1)^2} \underset{\substack{u=x^2+1 \\ du=2x\,dx \\ x\,dx=\frac{1}{2}du}}{=} \int \frac{\frac{1}{2}du}{u^2} = \frac{-1}{2u} + C = \frac{-1}{2(x^2+1)} + C$$

Dus $\int_0^\infty \frac{x\,dx}{(x^2+1)^2} = \lim_{A\to\infty} \int_0^A \frac{x\,dx}{(x^2+1)^2} = \lim_{A\to\infty} \left[\frac{-1}{2(x^2+1)}\right]_0^A =$

$$= \lim_{A\to\infty} \frac{-1}{2(A^2+1)} + \frac{1}{2} = \boxed{\frac{1}{2}}$$

b) $$\int (x+2)e^{2x}dx \underset{\substack{f(x)=x+2 \\ g'(x)=e^{2x} \\ g(x)=\frac{1}{2}e^{2x}}}{=} \frac{1}{2}(x+2)e^{2x} - \int 1\cdot\frac{1}{2}e^{2x}dx$$

$$= \frac{1}{2}(x+2)e^{2x} - \frac{1}{4}e^{2x} + C$$

$$\boxed{= \left(\frac{1}{2}x + \frac{3}{4}\right)e^{2x} + C}$$

2 $f(x,y) = x^3 + xy^2 - 3x^2 - 9x$

a) $\lim_{x\to\infty} f(x,0) = \lim_{x\to\infty} x^3 - 3x^2 - 9x = \infty$, $\lim_{x\to-\infty} f(x,0) = -\infty$

f neemt willekeurig grote en kleine waarden aan, dus f kan geen absoluut maximum of minimum aannemen.

b) $\frac{\partial f}{\partial x} = 3x^2 + y^2 - 6x - 9$, $\frac{\partial f}{\partial y} = 2xy$

$\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$ oplossen. $\frac{\partial f}{\partial y} = 0 \Rightarrow x = 0$ of $y = 0$.

$x=0$ of $y=0$ invullen in $\frac{\partial f}{\partial x} = 0$:

$y=0$: $3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x+1)(x-3) = 0 \Rightarrow x = -1$ of $x = 3$

Dit geeft de stationaire punten $(-1,0),(3,0)$

$x=0 \Rightarrow y^2-9=0 \Rightarrow y=\pm 3$. Dit geeft de stationaire punten $(0,3),(0,-3)$.

c) $A=\frac{\partial^2 f}{\partial x^2}=6x-6$, $B=\frac{\partial^2 f}{\partial y \partial x}=2y$, $C=\frac{\partial^2 f}{\partial y^2}=2x$, $H=AC-B^2$

	A	B	C	H	
$(-1,0)$	-12	0	-2	24	relatief maximum
$(3,0)$	12	0	6	72	relatief minimum
$(0,3)$	-6	6	0	-36	zadelpunt
$(0,-3)$	-6	-6	0	-36	zadelpunt.

d) $\frac{\partial f}{\partial x}(1,1)=3+1-6-9=-11$, $\frac{\partial f}{\partial y}(1,1)=2$, $f(1,1)=1+1-3-9=-10$

vgl raakvlak : $z-f(1,1)=\frac{\partial f}{\partial x}(1,1)(x-1)+\frac{\partial f}{\partial y}(1,1)(y-1)$

ofwel $z+10=-11(x-1)+2(y-1)$

$=-11x+2y+9$

$$\boxed{z=-11x+2y-1}$$

3 a) $z=3+4i$, $w=5-12i$ $|z|=\sqrt{3^2+4^2}=5$, $|w|=\sqrt{5^2+(-12)^2}=13$

$|z^2w|=|z|^2\cdot|w|=5^2\times 13=325$

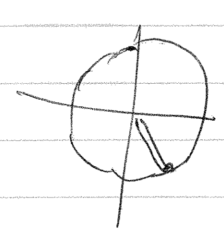
methode 2: $z^2=(3+4i)^2=(3+4i)(3+4i)=3\times 3+3\times 4i+4i\times 3+4^2\cdot i^2$

$=-7+24i$

$z^2w=(-7+24i)(5-12i)=-35+84i+120i-288i^2$

$=253+204i$

$|z^2w|=\sqrt{253^2+204^2}=325$.



b) $|9-9i|=\sqrt{9^2+(-9)^2}=\sqrt{162}=9\sqrt{2}$

$\frac{9-9i}{9\sqrt{2}}=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i=\cos(-\tfrac{1}{4}\pi)+i\sin(-\tfrac{1}{4}\pi)$

$9-9i=9\sqrt{2}\left(\cos(-\tfrac{1}{4}\pi)+i\sin(-\tfrac{1}{4}\pi)\right)$

$(9-9i)^{10}=(9\sqrt{2})^{10}\left(\cos(-\tfrac{10}{4}\pi)+i\sin(-\tfrac{10}{4}\pi)\right)$

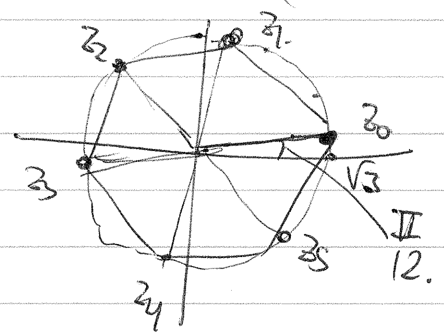
$= 9^{10} \times 2^{5} \left(\cos\left(-\tfrac{1}{2}\pi\right) + i\sin\left(-\tfrac{1}{2}\pi\right)\right) = \boxed{-9^{10} \times 2^{5} \cdot i}$

c) $27i = 27\left(\cos \tfrac{1}{2}\pi + i \sin \tfrac{1}{2}\pi\right)$

Oplossingen van $z^6 = 27i$: $\left(\tfrac{1}{6} \cdot \tfrac{1}{2}\pi = \tfrac{1}{12}\pi\right)$

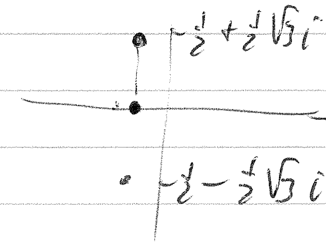
$z_k = \sqrt[6]{27}\left(\cos\left(\frac{\pi}{12} + \frac{2k\pi}{6}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{6}\right)\right)$ $(k = 0,1,2,3,4,5)$

$= \sqrt{3}\left(\cos\left(\frac{\pi}{12} + \frac{k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{k\pi}{3}\right)\right)$ $(k = 0,1,2,3,4,5)$



d) $(1+i)z^2 + (1+i)z + 1+i = 0 \xrightarrow{\text{deel door } 1+i} z^2 + z + 1 = 0$. $D = 1 - 4 = -3$

opl $z_{1,2} = \dfrac{-1 \pm \sqrt{3}\,i}{2}$.



4 a) Gebruik het vergelijkingskenmerk. Vergelijk $\frac{1}{n^2+3}$ met $\frac{1}{n^2}$

$\lim_{n\to\infty} \dfrac{1/(n^2+3)}{1/n^2} = \lim_{n\to\infty} \dfrac{n^2}{n^2+3} = \lim_{n\to\infty} \dfrac{1}{1+\frac{3}{n^2}} = 1$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergeert. Dus $\sum_{n=0}^{\infty} \frac{1}{n^2+3}$ $\boxed{\text{convergeert.}}$

b) De convergentiestraal van $\sum_{n=0}^{\infty} a_n x^n$ is
$\lim_{n\to\infty} \left|\dfrac{a_n}{a_{n+1}}\right|$ (mits de limiet bestaat)

In dit geval: $a_n = n^{17} \cdot 5^n$

Dus de convergentiestraal is

$$\lim_{n\to\infty} \frac{n^{17}\cdot 5^n}{(n+1)^{17}\cdot 5^{n+1}} = \lim_{n\to\infty} \frac{n^{17}}{(n+1)^{17}}\cdot \frac{1}{5} = \lim_{n\to\infty} \frac{1}{(1+\frac{1}{n})^{17}}\cdot\frac{1}{5} = \boxed{\frac{1}{5}}$$

c) $$\sum_{n=0}^{\infty}\left(\left(\frac{1}{2}\right)^n+\left(\frac{1}{3}\right)^n\right) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n = \frac{1}{1-\frac{1}{2}}+\frac{1}{1-\frac{1}{3}}$$

$$= \frac{1}{1/2}+\frac{1}{2/3} = 2+\frac{3}{2} = \boxed{\frac{7}{2}}$$

Hele stof.

(1) a) $$\lim_{x\to 0} \frac{\sqrt{1+x}-1-\frac{1}{2}x}{x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}}{2x} \overset{\frac{0}{0}}{=}$$

$$= \lim_{x\to 0} \frac{-\frac{1}{4}(1+x)^{-\frac{3}{2}}}{2} = \boxed{-\frac{1}{8}}$$

b) $$\lim_{x\to\infty} \sqrt{x^2+1}-\sqrt{x^2-1} = \lim_{x\to\infty} \frac{(\sqrt{x^2+1}-\sqrt{x^2-1})(\sqrt{x^2+1}+\sqrt{x^2-1})}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{(x^2+1)-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \boxed{0}$$

(2) $$f_c(x) = \begin{cases} c/x^2 & x<e \\ (\ln x^c)^2 & x\geq e \end{cases}$$

$$\lim_{x\downarrow e} f_c(x) = \lim_{x\downarrow e} (c\ln x)^2 = c^2, \quad \lim_{x\uparrow e} f_c(x) = \lim_{x\uparrow e} \frac{c}{x^2} = \frac{c}{e^2}$$

$$f_c(e) = (\ln e^c)^2 = c^2$$

f_c is continu in $x=e \iff \lim_{x\downarrow e} f_c(x) = f_c(e),\ \lim_{x\uparrow e} f_c(x) = f_c(e)$

$$\iff \frac{c}{e^2} = c^2 \iff c^2-\frac{c}{e^2}=0 \iff c\left(c-\frac{1}{e^2}\right)=0 \iff$$

$$\boxed{c=0 \text{ of } c=\frac{1}{e^2}}$$

(3) $f(x) = x^{4/3}$

n	$f^{(n)}(x)$	$f^{(n)}(8)$	$f^{(n)}(8)/n!$
0	$x^{4/3}$	$8^{4/3} = 16$	16
1	$\frac{4}{3}x^{1/3}$	$\frac{4}{3} \times 2 = \frac{8}{3}$	$\frac{8}{3}$
2	$\frac{4}{9}x^{-2/3}$	$\frac{4}{9} \times 2^{-2} = \frac{1}{9}$	$\frac{1}{18}$
3	$-\frac{8}{27}x^{-5/3}$	$-\frac{8}{27} \times 2^{-5} = -\frac{1}{108}$	$-\frac{1}{648}$
4	$\frac{40}{81}x^{-8/3}$		

$$P_3(x) = f(8) + \frac{f'(8)}{1!}(x-8) + \frac{f''(8)}{2!}(x-8)^2 + \frac{f'''(8)}{3!}(x-8)^3$$

$$= \boxed{16 + \frac{8}{3}(x-8) + \frac{1}{18}(x-8)^2 - \frac{1}{648}(x-8)^3}$$

$$E_3(x) = \frac{f^{(4)}(\theta)}{4!}(x-8)^4 = \frac{40}{81 \times 24}\theta^{-8/3}(x-8)^4 = \boxed{\frac{5}{243}\theta^{-8/3}(x-8)^4},$$

θ tussen 8 en x.

(4) $f(x) = \frac{x^6}{6x^7 - 1}$

a) f heeft een verticale asymptoot rond $x=a$, als de noemer in $x=a$ van f gelijk is aan 0.
In dit geval is dit in $\sqrt[7]{1/6}$. Dus $\boxed{x = \sqrt[7]{1/6}}$ is een verticale asymptoot van f.

$\lim_{x \downarrow \sqrt[7]{1/6}} f(x) = \infty$ (teller >0, noemer >0)

$\lim_{x \uparrow \sqrt[7]{1/6}} f(x) = -\infty$ (teller >0, noemer <0)

b) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^{-1}}{6 - \frac{1}{x^7}} = 0$, $\lim_{x\to-\infty} f(x) = 0$ (zelfde berekening)

Dus $\boxed{y=0}$ is een horizontale asymptoot van f zowel voor $x\to\infty$ als voor $x\to-\infty$.

c) $f'(x) = \dfrac{(6x^7-1)\cdot 6x^5 - x^6 \cdot 42x^6}{(6x^7-1)^2}$

$= \dfrac{36x^{12} - 6x^5 - 42x^{12}}{(6x^7-1)^2} = \dfrac{-6x^{12}-6x^5}{(6x^7-1)^2}$

$= \dfrac{-6x^5(x^7+1)}{(6x^7-1)^2}$ nulpunten: $x=0$, $x=-1$.

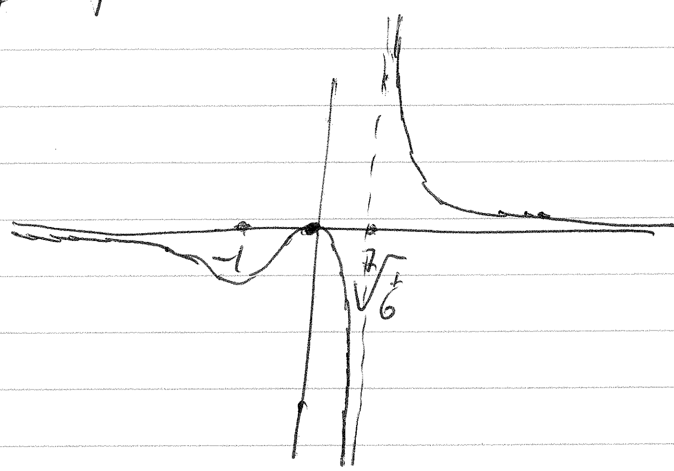
Tekenoverzicht f': $\quad - \;|_{-1}\; + \;|_{0}\; - \;|_{\sqrt[7]{1/6}}\; -$

f: $\quad \downarrow \; -1 \; \nearrow \; 0 \; \downarrow \; \sqrt[7]{1/6} \; \downarrow$

$f(0)=0$ relatief maximum (want $\lim_{x \downarrow \sqrt[7]{1/6}} f(x) = \infty$)

$f(-1) = \frac{1}{-7} = -\frac{1}{7}$ relatief minimum (want $\lim_{x \uparrow \sqrt[7]{1/6}} f(x) = -\infty$)

d)



5 a,b = 1 a,b van uitwerking deel 2.
6 a,b,c = 2 a,b,c "
7 a,b,c = 3 a,b,c "
8 a,b,c = 4 a,b,c "