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UITWERKING TENTAMEN CONTINUE WISKUNDE
10-1-2014

Deel 2

1) a) $\int x e^{-2x}\,dx \underset{\uparrow}{=} x\cdot(-\frac{1}{2}e^{-2x}) - \int(-\frac{1}{2}e^{-2x})\,dx$

$f(x) = x$
$g'(x) = e^{-2x}$
$g(x) = -\frac{1}{2}e^{-2x}$

$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$

b) Substitutie $u = \frac{1}{2}\pi\sqrt{x}$, $du = \frac{1}{2}\pi\cdot\frac{1}{2}x^{-\frac{1}{2}}\cancel{dx} = \frac{1}{4}\pi\,\frac{dx}{\sqrt{x}}$,

$\frac{dx}{\sqrt{x}} = \frac{1}{\frac{1}{4}\pi}\,du = \frac{4}{\pi}\,du$

Dus $\int \frac{\sin(\frac{1}{2}\pi\sqrt{x})}{\sqrt{x}}\,dx = \int \sin u\cdot\frac{4}{\pi}\,du = -\frac{4}{\pi}\cos u + C$

$= -\frac{4}{\pi}\cos(\frac{1}{2}\pi x) + C$

$\int_0^1 \frac{\sin(\frac{1}{2}\pi\sqrt{x})}{\sqrt{x}}\,dx = \lim_{\delta\downarrow 0}\int_\delta^1 \frac{\sin(\frac{1}{2}\pi\sqrt{x})}{\sqrt{x}}\,dx$

$= \lim_{\delta\downarrow 0}\left[-\frac{4}{\pi}\cos(\frac{1}{2}\pi x)\right]_\delta^1 = -\frac{4}{\pi}\cos(\frac{1}{2}\pi) + \frac{4}{\pi}\cos 0$

$= \boxed{\frac{4}{\pi}}$

2) $f(x,y) = 2x^3 + y^3 - 3x^2 - 12x - 3y$

a) $\lim_{x\to\infty} f(x,0) = \lim_{x\to\infty} 2x^3 - 3x^2 - 12x = \infty$

$\lim_{x\to-\infty} f(x,0) = \lim_{x\to-\infty} 2x^3 - 3x^2 - 12x = -\infty$

Dus f kan geen absolute maxima of minima aannemen.

b) $\frac{\partial f}{\partial x} = 6x^2 - 6x - 12, \quad \frac{\partial f}{\partial y} = 3y^2 - 3$

$\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0 \Leftrightarrow x^2 - x - 2 = 0$ en $y^2 - 1 = 0$

$\Leftrightarrow (x+1)(x-2) = 0$ en $(y+1)(y-1) = 0$

$\Leftrightarrow x = -1$ of $x = 2$ en $y = -1$ of $y = 1$

Dit geeft de 4 punten $(-1,-1), (2,-1), (-1,1), (2,1)$
en die voldoen allemaal aan $\frac{\partial f}{\partial x} = 0$ en $\frac{\partial f}{\partial y} = 0$

c) $\frac{\partial^2 f}{\partial x^2} = 12x - 6 = A, \quad \frac{\partial^2 f}{\partial y^2} = 6y = C, \quad \frac{\partial^2 f}{\partial y \partial x} = 0 = B, \quad H = AC - B^2$

	A	B	C	$H = AC - B^2$	conclusie
$(-1,-1)$	$-18 < 0$	0	-6	$108 > 0$	rel. maximum
$(2,-1)$	$18 > 0$	0	-6	$-108 < 0$	zadelpunt
$(-1,1)$	$-18 < 0$	0	6	$-108 < 0$	zadelpunt
$(2,1)$	$18 > 0$	0	6	$108 > 0$	rel. minimum

d) vgl raakvlak $z = \frac{\partial f}{\partial x}(1,0) \cdot (x-1) + \frac{\partial f}{\partial y}(1,0) \cdot (y-0) + f(1,0)$

$= -13 + (-12)(x-1) + (-3)y$

$= -13 - 12x + 12 - 3y$

$\boxed{z = -1 - 12x - 3y}$

3 a) $\frac{z}{w} = \frac{2+i}{7-i} = \frac{(2+i)(7+i)}{(7-i)(7+i)} = \frac{2 \cdot 7 + 2 \cdot i + i \cdot 7 + i^2}{7 \cdot 7 + 7 \cdot i - i \cdot 7 - i^2} = \boxed{\frac{13+9i}{50}}$

$\left|\frac{z}{w}\right| = \sqrt{\left(\frac{13}{50}\right)^2 + \left(\frac{9}{50}\right)^2} = \sqrt{\frac{169+81}{50^2}} = \frac{1}{50}\sqrt{250} = \frac{1}{50} \cdot 5\sqrt{10} = \boxed{\frac{1}{10}\sqrt{10}}$

b) $|4 + 4\sqrt{3}\,i| = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{16+48} = \sqrt{64} = 8 = r$

$4 + 4\sqrt{3}\,i = r(\cos\varphi + i\sin\varphi) \Rightarrow \cos\varphi = \frac{4}{r} = \frac{1}{2}, \quad \sin\varphi = \frac{4\sqrt{3}}{8} = \frac{1}{2}\sqrt{3}$

Dus we mogen $\varphi = \frac{1}{3}\pi$ nemen.

$(4 + 4\sqrt{3}\,i)^{10} = r^{10}(\cos 10\varphi + i \sin 10\varphi) = 8^{10}(\cos \frac{10}{3}\pi + i \sin \frac{10}{3}\pi)$

$= 2^{30}(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi) = 2^{30}(-\frac{1}{2} - \frac{1}{2}\sqrt{3}\,i) = \boxed{-2^{29}(1 + \frac{1}{2}\sqrt{3}\,i)}$

c) Schrijf $8\sqrt{2}(1-i)$ in de vorm $r(\cos\varphi + i\sin\varphi)$:

$r = |8\sqrt{2}(1-i)| = \sqrt{(8\sqrt{2})^2 + (-8\sqrt{2})^2} = \sqrt{128+128} = \sqrt{256} = 16$

$8\sqrt{2}(1-i) = 16\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

We mogen nemen: $\varphi = -\frac{1}{4}\pi$

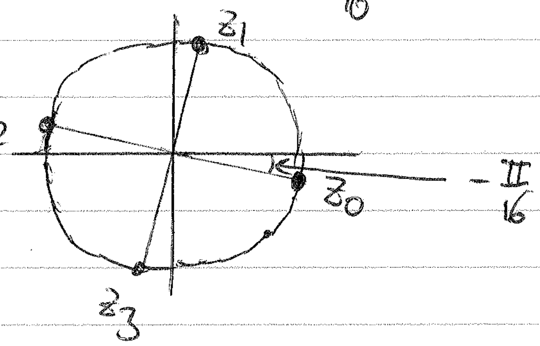
Dus $8\sqrt{2}(1-i) = 16\left(\cos(-\frac{1}{4}\pi) + i\sin(-\frac{1}{4}\pi)\right)$

Oplossingen van $z^4 = 8\sqrt{2}(1-i)$:

Deze liggen op een vierkant waarvan de omgeschreven cirkel straal $\sqrt[4]{16} = 2$ heeft, en waarvan een van de hoekpunten argument $\frac{1}{4}(-\frac{1}{4}\pi) = -\frac{1}{16}\pi$ heeft

$z_k = \sqrt[4]{16}\left(\cos\left(-\frac{\pi}{16} + \frac{2k\pi}{4}\right) + i\sin\left(-\frac{\pi}{16} + \frac{2k\pi}{4}\right)\right)$ $(k=0,1,2,3)$

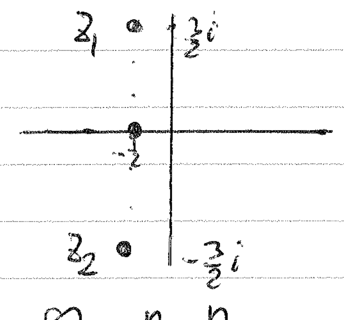
$= 2\left(\cos(-\frac{\pi}{16} + \frac{1}{2}k\pi) + i\sin(-\frac{\pi}{16} + \frac{1}{2}k\pi)\right)$ $(k=0,1,2,3)$



d) $2z^2+2z+5=0$ discriminant $D = 2^2 - 4\times2\times5 = -36$

Opl met abc-formule: $z_{1,2} = \frac{-b \pm i\sqrt{|D|}}{2a} = \frac{-2 \pm i\sqrt{36}}{4}$

$= \boxed{-\frac{1}{2} \pm \frac{3}{2}i}$



4) $f(x) = \sum_{n=0}^{\infty} \frac{2^n \cdot x^n}{n+9}$

a) $a_n = \frac{2^n}{n+9}$ Convergentiestraal $R_f = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$

$= \lim_{n\to\infty} \frac{2^n}{n+9} \Big/ \frac{2^{n+1}}{n+10} = \lim_{n\to\infty} \frac{2^n}{n+9} \cdot \frac{n+10}{2^{n+1}} = \lim_{n\to\infty} \frac{1}{2} \cdot \frac{n+10}{n+9} = \boxed{\frac{1}{2}}$

b) $f(\tfrac{1}{2}) = \sum_{n=0}^{\infty} \frac{2^n \cdot (\frac{1}{2})^n}{3n+9} = \sum_{n=0}^{\infty} \frac{1}{3n+9}$

Gebruik het vergelijkingskenmerk met $a_n = \frac{1}{3n+9}$, $b_n = \frac{1}{n}$.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1/3n+9}{1/n} = \lim_{n\to\infty} \frac{n}{3n+9} = \lim_{n\to\infty} \frac{1}{3+9/n} = \frac{1}{3}.$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ divergeert. Dus $\sum_{n=0}^{\infty} a_n = f(\tfrac{1}{2})$ $\boxed{\text{divergeert}}$

c) $f(x)$ convergeert voor $-R_f < x < R_f$ en divergeert als $x > R_f$ of $x < -R_f$

$R_f = \frac{1}{2}$. Dus $f(0{,}49)$ convergeert en $f(0{,}51)$ divergeert.

<u>Hele stof</u>

1) $\lim_{x\to 0} \frac{\ln(1+2x)-2x}{x^2} \overset{0/0}{=} \lim_{x\to 0} \frac{\frac{2}{1+2x}-2}{2x} \overset{0/0}{=}$

$\lim_{x\to 0} \frac{\frac{-4}{(1+2x)^2}}{2} = \boxed{-2}$

$\lim_{x\to\infty} \frac{e^{x+1}+x}{e^x-1} = \lim_{x\to\infty} \frac{e^{x+1}/e^x + \frac{x}{e^x}}{1-\frac{1}{e^x}} = \lim_{x\to\infty} \frac{e + \overset{\to 0}{\frac{x}{e^x}}}{1-\underset{\to 0}{\frac{1}{e^x}}}$

$= \boxed{e}$

2) $\lim_{x\downarrow 1} f(x) = \lim_{x\downarrow 1} \sqrt{4x} = \sqrt{4} = 2$

$\lim_{x\uparrow 1} f(x) = \lim_{x\uparrow 1} \frac{1}{\cos(\frac{1}{3}\pi x)} = \frac{1}{\cos(\frac{1}{3}\pi)} = \frac{1}{\frac{1}{2}} = 2$

Dus $\lim_{x\to 1} f(x)$ bestaat en is gelijk aan 2

$f(1) = 1$. Dus $\lim_{x\to 1} f(x) \neq f(1)$, f is niet continu in $x=1$.

3)

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$f^{(n)}(0)/n!$
0	$(x+1)^{1/6}$	1	1
1	$\frac{1}{6}(x+1)^{-5/6}$	$\frac{1}{6}$	$\frac{1}{6}$
2	$-\frac{5}{36}(x+1)^{-\frac{11}{6}}$	$-\frac{5}{36}$	$-\frac{5}{72}$
3	$\frac{55}{216}(x+1)^{-\frac{17}{6}}$	$\frac{55}{216}$	$\frac{55}{1296}$
4	$\frac{-935}{1296}(x+1)^{-\frac{23}{6}}$		

$P_3(x) = 1 + \frac{1}{6}x - \frac{5}{72}x^2 + \frac{55}{1296}x^3$

$E_3(x) = \frac{f^{(4)}(\xi)}{4!}x^4$ ξ tussen 0 en x

$= \frac{-935(1+\xi)^{-29/6}}{31104} \cdot x^4$

4) $f(x) = \frac{x}{x^2-3x+2} = \frac{x}{(x-1)(x-2)}$

a) Verticale asymptoten: $x=1$, $x=2$

$\lim_{x \downarrow 2} f(x) = \infty$, $\lim_{x \uparrow 2} f(x) = -\infty$, $\lim_{x \downarrow 1} f(x) = -\infty$, $\lim_{x \uparrow 1} f(x) = \infty$

b) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1/x}{1 - \frac{3}{x} + \frac{2}{x^2}} = 0$

$\lim_{x \to -\infty} f(x) = 0$ zelfde berekening

Dus $y=0$ is een horizontale asymptoot van f zowel voor $x \to \infty$ als $x \to -\infty$

c) $f'(x) = \frac{x^2-3x+2 - x(2x-3)}{(x^2-3x+2)^2} = \frac{-x^2-3x+2+3x}{(x^2-3x+2)^2}$

$= \frac{-x^2+2}{(x^2-3x+2)^2}$

⑥

f': $-$ | $+$ | NG | $+$ | $-$ | NG | $-$

$-\sqrt{2}$ $\sqrt{2}$ 2

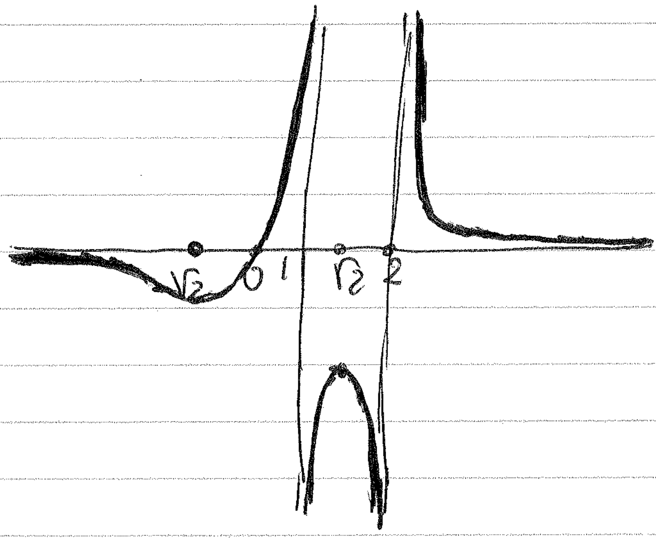
f

$x = -\sqrt{2}$ $f(-\sqrt{2}) = \dfrac{-\sqrt{2}}{(-\sqrt{2})^2 - 3(-\sqrt{2}) + 2} = \dfrac{-\sqrt{2}}{4 + 3\sqrt{2}} \approx -0{,}1715$

relatief minimum (want $\lim\limits_{x \downarrow -1} f(x) = -\infty$)

$x = \sqrt{2}$ $f(\sqrt{2}) = \dfrac{\sqrt{2}}{(\sqrt{2})^2 - 3\sqrt{2} + 2} = \dfrac{\sqrt{2}}{4 - 3\sqrt{2}} \approx -5{,}8284$

relatief maximum (want kleiner dan minimum)



5 a,b, 6 a,b,c, 7 a,b,c, 8 a,b,c

= 1 a,b, 2 a,b,c, 3 a,b,c, 4 a,b,c
van tweede deeltentamen.