UITWERKING CONTINUE WISKUNDE
HERKANSING DEEL 2, 13-3-2015

1 a) $\int \sqrt{2+\sin x}\,\cos x\,dx \underset{\substack{u=2+\sin x \\ du=\cos x\cdot dx}}{=} \int \sqrt{u}\,du = \frac{2}{3}u^{3/2}+C$

$= \boxed{\frac{2}{3}(2+\sin x)^{3/2}+C}$

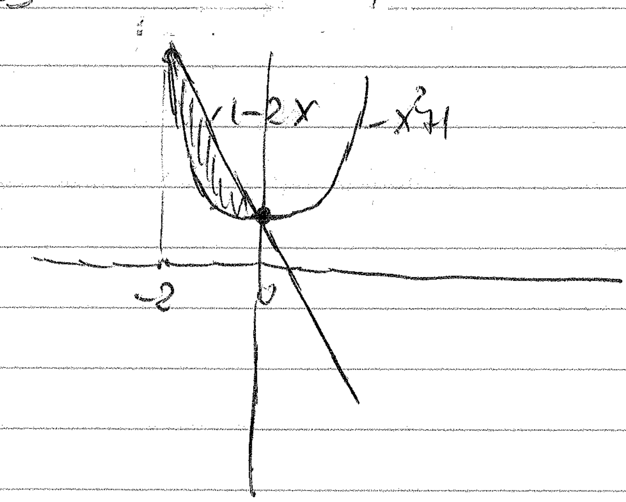
b) $\int_0^\infty (x+1)e^{-x}dx = \lim_{A\to\infty}\int_0^A (x+1)e^{-x}dx \underset{\substack{f(x)=x+1 \\ g'(x)=e^{-x} \\ g(x)=-e^{-x}}}{=}$

$= \lim_{A\to\infty}\left(\left[-(x+1)e^{-x}\right]_0^A - \int_0^A -e^{-x}(x+1)'\,dx \right)$

$= \lim_{A\to\infty}\left(-(A+1)e^{-A}+1+\int_0^A e^{-x}dx \right) = \lim_{A\to\infty}\left(-(A+1)e^{-A}+1+\left[-e^{-x}\right]_0^A \right)$

$= \lim_{A\to\infty}\left(-(A+1)e^{-A}+1+(-e^{-A}+1) \right) = 2 - \lim_{A\to\infty}(A+2)e^{-A} = \boxed{2}$

c)



snijpunten van de grafieken van $f(x)=x^2+1$ en $g(x)=1-2x$

$x^2+1=1-2x \Leftrightarrow$
$x^2+2x=0 \Leftrightarrow x(x+2)=0$
$\Leftrightarrow x=0$ of $x=-2$

oppervlakte: $\int_{-2}^{0}\left((1-2x)-(x^2+1)\right)dx = \int_{-2}^{0}(-2x-x^2)\,dx = \left[-x^2-\frac{1}{3}x^3\right]_{-2}^{0}$

$= 0-\left(-4+\frac{8}{3}\right) = \boxed{\frac{4}{3}}$

2 a) $f(x,y) = 2x^3 - 6xy + 3y^2 - 12y$

$\frac{\partial f}{\partial x} = 6x^2 - 6y$, $\frac{\partial f}{\partial y} = -6x + 6y - 12$

stationaire punten: $\frac{\partial f}{\partial x} = 0$ en $\frac{\partial f}{\partial y} = 0 \Leftrightarrow$

$y = x^2$ en $y = x+2 \Leftrightarrow x^2 = x+2$ en $y = x^2$

$\Leftrightarrow x^2 - x - 2 = 0$ en $y = x^2 \Leftrightarrow (x+1)(x-2) = 0$ en $y = x^2$

$\Leftrightarrow (x,y) = (-1,1)$ of $(2,4)$.

b) $\frac{\partial^2 f}{\partial x^2} = 12x$, $\frac{\partial^2 f}{\partial y \partial x} = -6$, $\frac{\partial^2 f}{\partial y^2} = 6$

	$A = \frac{\partial^2 f}{\partial x^2}$	$B = \frac{\partial^2 f}{\partial y \partial x}$	$C = \frac{\partial^2 f}{\partial y^2}$	$H = AC - B^2$	
$(-1,1)$	-12	-6	6	$-108 < 0$	zadelpunt
$(2,4)$	$24 > 0$	-6	6	$24 \times 6 - 36 > 0$	minimum.

f neemt in $(2,4)$ een <u>relatief</u> minimum aan, want

$\lim_{\substack{x \to -\infty \\ y = 0}} f(x,y) = \lim_{x \to -\infty} 2x^3 = -\infty$

c) Vergelijking raakvlak: $z = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1)$

$f(1,1) = 2 - 6 + 3 - 12 = -13$

$\frac{\partial f}{\partial x}(1,1) = 6.1 - 6.1 = 0$, $\frac{\partial f}{\partial y}(1,1) = -6.1 + 6.1 - 12 = -12$

vgl. $z = -13 + 0.(x-1) - 12(y-1) = -13 - 12y + 12 \Leftrightarrow$

$\boxed{z = -12y - 1}$

3 a) $(3+4i)z = 3-4i \Leftrightarrow z = \frac{3-4i}{3+4i} \Leftrightarrow$

$z = \frac{(3-4i)(3-4i)}{3^2+4^2} = \frac{3\times 3 - 3\times 4i - 4i\times 3 - 4i\cdot(-4i)}{25}$

$= \frac{9-24i+16i^2}{25} = \boxed{\frac{-7-24i}{25}}$

$|z| = \sqrt{\left(\frac{-7}{25}\right)^2 + \left(\frac{-24}{25}\right)^2} = \sqrt{\frac{49+576}{625}} = \sqrt{\frac{625}{625}} = \boxed{1}$

of. $|z| = \left|\frac{3-4i}{3+4i}\right| = \frac{|3-4i|}{|3+4i|} = \frac{\sqrt{3^2+(-4)^2}}{\sqrt{3^2+4^2}} = 1$

b) Schrijf $\sqrt{3}+i = r(\cos\varphi + i\sin\varphi)$ $r = |\sqrt{3}+i|$

Er geldt: $r = \sqrt{(\sqrt{3})^2+1} = \sqrt{3+1} = 2$

$\cos\varphi = \frac{1}{2}\sqrt{3}$, $\sin\varphi = \frac{1}{2}$. Dus

$\sqrt{3}+i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$,

$(\sqrt{3}+i)^{120} = 2^{120}\left(\cos\frac{120}{6}\pi + i\sin\frac{120}{6}\pi\right) = 2^{120}(\cos 20\pi + i\sin 20\pi)$

$= \boxed{2^{120}}$.

c) Schrijf $3\sqrt{2} - 3\sqrt{2}\,i = r(\cos\varphi + i\sin\varphi)$

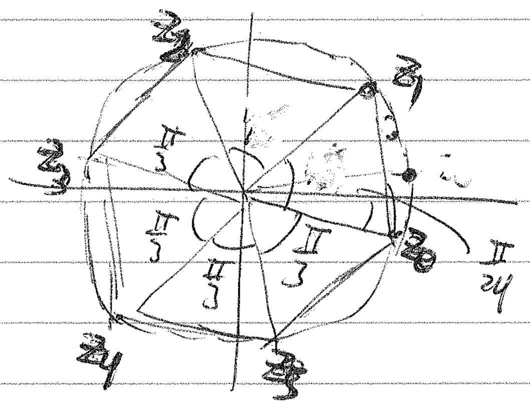
$r = \sqrt{(3\sqrt{2})^2 + (-3\sqrt{2})^2} = \sqrt{18+18} = \sqrt{36} = 6$

$\cos\varphi = \frac{3\sqrt{2}}{6} = \frac{1}{2}\sqrt{2}$, $\sin\varphi = \frac{-3\sqrt{2}}{6} = -\frac{1}{2}\sqrt{2}$

$3\sqrt{2} - 3\sqrt{2}\,i = 6\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$.

Oplossingen van $z^6 = 3\sqrt{2} - 3\sqrt{2}i$

$$z = \sqrt[6]{6}\left(\cos\left(-\frac{\pi}{24} + \frac{2k\pi}{6}\right) + i\sin\left(-\frac{\pi}{24} + \frac{2k\pi}{6}\right)\right) \quad (k=0,1,2,3,4,5)$$



d) $(2+i)z^2 + (4+2i)z + 4+2i = 0 \Leftrightarrow$ (deel door $2+i$)

$z^2 + 2z + 2 = 0 \Leftrightarrow z_{1,2} = \frac{-2 \pm i\sqrt{4}}{2} = \boxed{-1 \pm i}$

$D = 2^2 - 4 \times 2 = -4$

4 a) Gebruik het quotientkenmerk:

$a_n \geq 0$, $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1 \Rightarrow \sum_{n=0}^{\infty} a_n$ convergent

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1 \Rightarrow \sum_{n=0}^{\infty} a_n$ divergent

Pas dit toe met $a_n = \frac{n^3}{3^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \Big/ \frac{n^3}{3^n} = \lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^3 \cdot \frac{1}{3}$$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^3 \cdot \frac{1}{3} = \frac{1}{3} < 1$$

Dus $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$ $\boxed{\text{convergeert.}}$

b) $$\sum_{n=0}^{\infty} \frac{4^n - 3^n}{5^n} = \sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n - \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{1}{1-\frac{4}{5}} - \frac{1}{1-\frac{3}{5}}$$

$$= \frac{1}{1/5} - \frac{1}{2/5} = 5 - \frac{5}{2} = \boxed{\frac{5}{2}}$$

UITWERKING CONTINUE WISKUNDE
HERKANSING HELE STOF, 13-3-2015

① a) $\lim\limits_{x \to \frac{1}{2}\pi} \dfrac{(x-\frac{1}{2}\pi)^2}{1-\sin x} \overset{0}{\underset{0}{=}} \lim\limits_{x \to \frac{1}{2}\pi} \dfrac{2(x-\frac{1}{2}\pi)}{-\cos x} \overset{0}{\underset{0}{=}}$

$= \lim\limits_{x \to \frac{1}{2}\pi} \dfrac{2}{\sin x} = \boxed{2}$

b) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x} + \ln x}{\sqrt{x} + x^{2/5}} = \lim\limits_{x \to \infty} \dfrac{1 + \frac{\ln x}{x^{1/2}}}{1 + x^{1/4 - \frac{1}{2}}} = \lim\limits_{x \to \infty} \dfrac{1 + \overbrace{\frac{\ln x}{x^{1/2}}}^{\to 0}}{1 + \underbrace{x^{-1/4}}_{\to 0}} = 1.$

② Merk op: $f(0) = \cos 0 = 1$, $f(1) = \cos \frac{1}{2}\pi = 0$

a) $\lim\limits_{x \downarrow 1} f(x) = \lim\limits_{x \downarrow 1} \ln cx = \ln c$

$\lim\limits_{x \uparrow 1} f(x) = \lim\limits_{x \uparrow 1} \cos \frac{1}{2}\pi x = 0$

Er moet gelden: $\lim\limits_{x \downarrow 1} f(x) = f(1)$, $\lim\limits_{x \uparrow 1} f(x) = f(1)$. Dit geldt alleen wanneer $\ln c = 0$, dwz $\boxed{c = 1}$

b) $\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{x \downarrow 0} \cos \frac{1}{2}\pi x = 1$

$\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \uparrow 0} (x+d)^2 = d^2$

Er moet gelden: $\lim\limits_{x \downarrow 0} f(x) = f(0)$, $\lim\limits_{x \uparrow 0} f(x) = f(0)$

Dit geldt alleen voor $d^2 = 1$ dwz $\boxed{d = 1 \text{ of } d = -1}$

3 a)

n	$f^{(n)}(x)$	$f^{(n)}(16)$	$f^{(n)}(16)/n!$
0	$x^{1/4}$	2	2
1	$\frac{1}{4}x^{-3/4}$	$\frac{1}{4}(16)^{-3/4} = \frac{1}{4}\times 2^{-3} = 2^{-5}$	2^{-5}
2	$\frac{1}{4}\times(-\frac{3}{4})x^{-7/4} = -\frac{3}{16}x^{-7/4}$	$-\frac{3}{16}(16)^{-7/4} = -\frac{3}{16}\cdot 2^{-7} = -3\times 2^{-11}$	-3×2^{-12}
3	$-\frac{3}{16}\times -\frac{7}{4}x^{-11/4} = \frac{21}{64}x^{-11/4}$		

$$P_2(x) = f(16) + f'(16)(x-16) + \frac{f^{(2)}(16)}{2!}(x-16)^2$$

$$= 2 + 2^{-5}(x-16) - 3\times 2^{-12}(x-16)^2$$

b) $$E_2(x) = \frac{f^{(3)}(\xi)}{3!}(x-16)^3 = \frac{\frac{21}{64}\xi^{-11/4}}{6}(x-16)^3 = \frac{7}{128}\xi^{-11/4}(x-16)^3$$

met ξ tussen 16 en x

c) Omdat $16<\xi<17$ geldt:

$$|E_2(17)| < \frac{7}{128}16^{-11/4}(17-16)^3 = \frac{7}{128}\cdot 2^{-11} = \frac{7}{2^7}\times 2^{-11} = 7\times 2^{-18}$$

(4) $f(x) = \frac{1}{e^x - 1}$

a) f heeft verticale asymptoten $x = a$ voor die waarden van a waarvoor de noemer 0 is.
Er geldt: $e^x - 1 = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$

$\lim_{x \downarrow 0} f(x) = \infty$ (teller > 0, noemer > 0)

$\lim_{x \uparrow 0} f(x) = -\infty$ (teller < 0, noemer > 0)

b) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1}{e^x - 1} = 0$ want $e^x \to \infty$ als $x \to \infty$

Dus $\boxed{y = 0}$ is een horizontale asymptoot van f voor $x \to \infty$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1}{\underset{\downarrow}{\overset{}{e^x}}_{0} - 1} = \frac{1}{-1} = -1$ want $\lim_{x \to -\infty} e^x = 0$

Dus $\boxed{y = -1}$ is een horizontale asymptoot van f voor $x \to -\infty$

c) $f'(x) = \frac{(e^x - 1) \cdot 0 - 1 \cdot e^x}{(e^x - 1)^2} = \frac{-e^x}{(e^x - 1)^2}$

Tekenoverzicht f':

f'	$-$	NG	$-$
		0	
f	$\searrow$		$\searrow$

f is dalend voor elke x.
f heeft dus geen extremen

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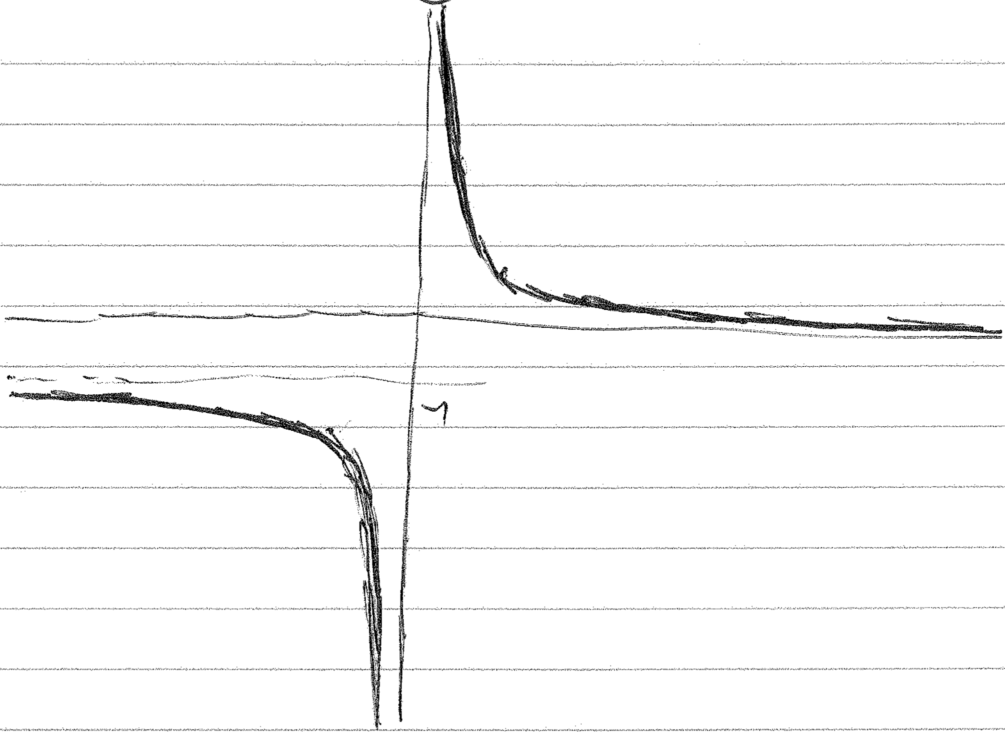

(5) Onderdeel a,b van deel 2, opgave 1.

(6) Onderdeel a,b van deel 2, opgave 2

(7) Onderdeel a,b,c van deel 2, opgave 3

(8) Deel 2, opgave 4