UITWERKING HERKANSING CONTINUE WISKUNDE 2, 6 juli 2017

1) a) De inhoud van het gevraagde omwentelingslichaam is

$$\int_{\frac{1}{2}\pi}^{\frac{5}{3}\pi} \pi f(x)^2 dx = \int_{\frac{1}{2}\pi}^{\frac{5}{3}\pi} \pi(1+\sin x)\, dx = \left[\pi(x-\cos x)\right]_{\frac{1}{2}\pi}^{\frac{5}{3}\pi}$$

$$= \pi\left(\tfrac{5}{3}\pi - \tfrac{1}{2} - \tfrac{1}{2}\pi + 0\right) = \boxed{\pi\left(\tfrac{7}{6}\pi - \tfrac{1}{2}\right)}$$

b) $$\int \sqrt{x}\ln x\, dx = \int \ln x . \sqrt{x}\, dx \underset{\uparrow}{=} (\ln x)\tfrac{2}{3}x^{3/2} - \int \ln' x : \tfrac{2}{3}x^{3/2} dx$$

$f(x) = \ln x$
$g'(x) = \sqrt{x}$
$g(x) = \tfrac{2}{3}x^{3/2}$

$$= \tfrac{2}{3}x^{3/2}\ln x - \int \tfrac{1}{x}\cdot\tfrac{2}{3}x^{3/2} dx$$

$$= \tfrac{2}{3}x^{3/2}\ln x - \tfrac{2}{3}\int x^{1/2} dx$$

$$= \boxed{\tfrac{2}{3}x^{3/2}\ln x - \tfrac{4}{9}x^{3/2} + C}$$

c) $$\int_0^\infty \frac{e^x}{(e^x+1)^2} dx = \lim_{B\to\infty} \int_0^B \frac{e^x}{(e^x+1)^2} dx \underset{\uparrow}{=} \lim_{B\to\infty} \int_2^{e^B+1} \frac{du}{u^2}$$

$u = e^x + 1$
$du = e^x dx$

$$= \lim_{B\to\infty} \left[-\tfrac{1}{u}\right]_2^{e^B+1} = \lim_{B\to\infty} -\frac{1}{e^B+1} + \frac{1}{2} = \boxed{\tfrac{1}{2}}$$

2) $f(x,y) = x^3y^2 - x^2 - y^2 + x$

a) $\lim_{\substack{x\to\infty \\ y=0}} f(x,y) = \lim_{x\to\infty} -x^2+x = -\infty \Rightarrow f$ neemt geen absoluut minimum aan.

$\lim_{\substack{x\to\infty \\ y=1}} f(x,y) = \lim_{x\to\infty} x^3 - x^2 - 1 + x = \infty \Rightarrow f$ neemt geen absoluut maximum aan.

b) $\frac{\partial f}{\partial x} = 3x^2y^2 - 2x + 1$, $\frac{\partial f}{\partial y} = 2x^3y - 2y = 2y(x^3 - 1)$

(x, y) stationair punt van $f \Leftrightarrow \frac{\partial f}{\partial x} = 0$ en $\frac{\partial f}{\partial y} = 0$

$\frac{\partial f}{\partial y} = 0 \Leftrightarrow x = 1$ of $y = 0$

$y = 0$ combineren met $\frac{\partial f}{\partial x} = 0$ geeft $-2x + 1 = 0$, m.a.w. $x = \frac{1}{2}$

Dus $(\frac{1}{2}, 0)$ is een stationair punt van f

$x = 1$ combineren met $\frac{\partial f}{\partial x} = 0$ geeft $3y^2 - 1 = 0$ m.a.w. $y = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{3}\sqrt{3}$

dit geeft de stationaire punten $(\frac{1}{2}, 0)$, $(1, \frac{1}{3}\sqrt{3})$, $(1, -\frac{1}{3}\sqrt{3})$

c) $\frac{\partial^2 f}{\partial x^2} = 6xy^2 - 2 = A$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 6x^2y = B$, $\frac{\partial^2 f}{\partial y^2} = 2x^3 - 2$, $H = AC - B^2$

	A	B	C	H	conclusie
$(\frac{1}{2}, 0)$	$-2 < 0$	0	$-\frac{7}{4}$	$\frac{7}{2} > 0$	relatief maximum
$(1, \frac{1}{3}\sqrt{3})$	0	$2\sqrt{3}$	0	$-12 < 0$	zadelpunt
$(1, -\frac{1}{3}\sqrt{3})$	0	$-2\sqrt{3}$	0	$-12 < 0$	zadelpunt.

d) Vergelijking raakvlak in $(2, 3, f(2,3)) = (2, 3, 61)$:

$\frac{\partial f}{\partial x}(2,3) = 3 \times 2^2 \times 3^2 - 2 \times 2 + 1 = 108 - 4 + 1 = 105$,

$\frac{\partial f}{\partial y}(2,3) = 2 \times 2^3 \times 3 - 2 \times 3 = 48 - 6 = 42$,

$z = f(2,3) + \frac{\partial f}{\partial x}(2,3)(x-2) + \frac{\partial f}{\partial y}(2,3)(y-3)$

$= \boxed{61 + 105(x-2) + 42(y-3)}$

③

③ a) $z = e^{5+\pi i/3} = e^5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = e^5\left(\frac{1}{2} + i\cdot\frac{1}{2}\sqrt{3}\right)$

$= \boxed{\frac{1}{2}e^5 + \frac{1}{2}\sqrt{3}e^5 . i}$

$|z| = e^5$ (algemeen: $|e^{a+bi}| = e^a$).

b) $|4+4i| = \sqrt{4^2+4^2} = \sqrt{32} = 2^{5/2} = 4\sqrt{2}$

$4+4i = 4\sqrt{2}.\left(\frac{4}{4\sqrt{2}} + \frac{4}{\sqrt{2}}i\right) = 4\sqrt{2}.\left(\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}\,i\right)$

$= 4\sqrt{2}\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)$

$(4+4i)^{103} = \left(2^{\frac{5}{2}}\right)^{103}.\left(\cos\frac{103}{4}\pi + i\sin\frac{103}{4}\pi\right)$ $\qquad \frac{103}{4} = 25\frac{3}{4}$

$= 26 - \frac{1}{4}\pi$

$= 2^{515/2}\left(\cos\left(26\pi - \frac{1}{4}\pi\right) + i\sin\left(26\pi - \frac{1}{4}\pi\right)\right)$

$= 2^{515/2}\left(\cos\left(-\frac{1}{4}\pi\right) + i\sin\left(-\frac{1}{4}\pi\right)\right) = 2^{\frac{515}{2}}\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}\,i\right)$

$= \boxed{2^{257}(1-i)}$

c) Stel $z+i = w$.

$2(z+i)^2 + 8(z+i) + 10 = 0 \Leftrightarrow 2w^2 + 8w + 10 = 0 \Leftrightarrow w^2 + 4w + 5 = 0$

discriminant $D = 4^2 - 4\times5 = -4$.

oplossingen: $w = \frac{-4 \pm \sqrt{4}.\,i}{2} = -2 \pm i$

$\boxed{z = w - i = -2,\ -2-2i}$

d) $243i = 243\left(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi\right)$

oplossingen: $z = \sqrt[5]{243}.\left(\cos\left(\frac{1}{10}\pi + \frac{2k\pi}{5}\right) + i\sin\left(\frac{1}{10}\pi + \frac{2k\pi}{5}\right)\right)$

$= \boxed{3\left(\cos\left(\frac{\pi}{10} + \frac{2k\pi}{5}\right) + i\sin\left(\frac{\pi}{10} + \frac{2k\pi}{5}\right)\right) \quad k = 0, 1, 2, 3, 4}$

4 a) Gebruik het vergelijkingskenmerk:
Stel $a_k \geq 0$, $b_k \geq 0$, $\lim_{k\to\infty} \frac{a_k}{b_k} = l$ met $0 < l < \infty$

Dan $\sum_{k=1}^{\infty} a_k$ convergent $\Leftrightarrow$ $\sum_{k=1}^{\infty} b_k$ convergent.

Er geldt voor k groot: $\frac{3k^3+7}{2k^5-1} \approx \frac{3k^3}{2k^5} \approx \frac{3}{2} \cdot k^{-2}$.

Pas het vergelijkingskenmerk toe met $a_k = \frac{3k^3+7}{2k^5-1}$, $b_k = k^{-2}$

Dan $\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{3k^3+7}{2k^5-1} \Big/ \frac{1}{k^2} = \lim_{k\to\infty} \frac{(3k^3+7)k^2}{2k^5-1}$

$= \lim_{k\to\infty} \frac{3k^5+7k^2}{2k^5-1} = \lim_{k\to\infty} \frac{3+7k^{-3}}{2-k^{-5}} = \frac{3}{2}$.

$\sum_{k=1}^{\infty} k^{-2}$ convergeert. Dus $\sum_{k=1}^{\infty} \frac{3k^3+7}{2k^5-1}$ $\boxed{\text{convergeert}}$

b) Gebruik het quotiëntkenmerk:
Stel $a_k \geq 0$, $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = l$ bestaat.

Als $l < 1$ dan is $\sum_{k=1}^{\infty} a_k$ convergent, als $l > 1$ dan is $\sum_{k=1}^{\infty} a_k$ divergent

Pas dit toe met $a_k = \frac{k}{\sqrt[3]{k!}}$

Er geldt: $\frac{a_{k+1}}{a_k} = \frac{k+1}{\sqrt[3]{(k+1)!}} \Big/ \frac{k}{\sqrt[3]{k!}} = \frac{k+1}{\sqrt[3]{(k+1)!}} \cdot \frac{\sqrt[3]{k!}}{k} = \frac{k+1}{k} \sqrt[3]{\frac{k!}{(k+1)!}}$

$= \frac{k+1}{k} \sqrt[3]{\frac{1 \cdot 2 \cdots k}{1 \cdot 2 \cdots k(k+1)}} = \frac{k+1}{k} \sqrt[3]{\frac{1}{k+1}}$

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 0$ Dus $\sum_{k=1}^{\infty} a_k$ is $\boxed{\text{convergent}}$