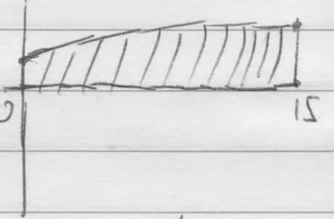
UITWERKING HERKANSING CONTINUE WISKUNDE 2
4 JULI 2019

1 a). De functie $f(x)=\sqrt[4]{x+1}$ is stijgend op $[0,15]$



gebied

Inhoud omwentelingslichaam

$$\int_0^{15} \pi f(x)^2\,dx = \int_0^{15} \pi\sqrt{x+1}\,dx = \left[\pi\,\tfrac{2}{3}(x+1)^{3/2}\right]_0^{15}$$

$$= \pi\cdot\tfrac{2}{3}\,16^{3/2} - \pi\cdot\tfrac{2}{3} = \pi\cdot\tfrac{2}{3}(64-1) = \pi\cdot\tfrac{2}{3}\cdot 63 = \boxed{42\pi}$$

b) $\int(\sqrt{x}+\sqrt[3]{x})\sin(8x^{3/2}+9x^{4/3})\,dx =$

$u = 8x^{3/2}+9x^{4/3}$

$du = (12\sqrt{x}+12\sqrt[3]{x})\,dx$

$(\sqrt{x}+\sqrt[3]{x})\,dx = \frac{1}{12}\,du$

$$= \int \sin u\cdot\tfrac{1}{12}\,du = -\tfrac{1}{12}\cos u + C = \boxed{-\tfrac{1}{12}\cos(8x^{3/2}+9x^{4/3})+C}$$

c) $\int x^3\ln x\,dx = (\ln x)\cdot\frac{1}{4}x^4 - \int\frac{1}{4}x^4\ln' x\,dx = \frac{1}{4}x^4\ln x - \int\frac{1}{4}x^3\,dx$

$f(x)=\ln x$

$g'(x)=x^3$

$g(x)=\frac{1}{4}x^4$

$$= \boxed{\tfrac{1}{4}x^4\ln x - \tfrac{1}{16}x^4 + C}$$

$$\int_0^1 x^3\ln x\,dx = \lim_{\delta\downarrow 0}\int_\delta^1 x^3\ln x\,dx = \lim_{\delta\downarrow 0}\left[\tfrac{1}{4}x^4\ln x - \tfrac{1}{16}x^4\right]_\delta^1$$

$$= \lim_{\delta\downarrow 0}\Big(0 - \tfrac{1}{16} - \underbrace{\tfrac{1}{4}\delta^4\ln\delta}_{\downarrow\,0} + \underbrace{\tfrac{1}{16}\delta^4}_{\downarrow\,0}\Big) = \boxed{-\tfrac{1}{16}}$$

Standaardlimiet: $\lim_{t\downarrow 0} t^a\ln t = 0$ als $a>0$.

(2) $f(x,y) = x^7 + 7xy + \frac{7}{2}y^2$

a) $\frac{\partial f}{\partial x} = 7x^6 + 7y$, $\frac{\partial f}{\partial y} = 7x + 7y$

stationaire punten: $\frac{\partial f}{\partial x} = 0$ en $\frac{\partial f}{\partial y} = 0 \Leftrightarrow x^6 = -y$ en $x = -y$

$\Leftrightarrow x^6 = x$ en $x = -y \Leftrightarrow x^6 - x = 0$ en $x = -y$

$\Leftrightarrow x(x^5 - 1) = 0$ en $x = -y \Leftrightarrow x = 0$ of $x = 1$ en $x = -y$

$\Leftrightarrow (x,y) = (0,0)$ of $(1,-1)$.

b) $\frac{\partial^2 f}{\partial x^2} = 42x^5 = A$, $\frac{\partial^2 f}{\partial y \partial x} = 7 = B$, $\frac{\partial^2 f}{\partial y^2} = 7 = C$, $H = AC - B^2$

	A	B	C	H	
(0,0)	0	7	7	$-49 < 0$	zadelpunt
(1,−1)	$42 > 0$	7	7	$42 \cdot 7 - 7^2 > 0$	minimum

$\lim_{\substack{x \to -\infty \\ y = 0}} f(x,y) = \lim_{x \to -\infty} x^7 = -\infty$

Dus het minimum is $\boxed{\text{relatief}}$

c) vergelijking raakvlak

$f(1, \frac{1}{7}) = 1 + 7 + \frac{7}{14} = 8 + \frac{1}{2} = \frac{17}{2}$, $\frac{\partial f}{\partial x}(1, \frac{1}{7}) = 8$, $\frac{\partial f}{\partial y}(1, \frac{1}{7}) = 8$

$z = f(1, \frac{1}{7}) + \frac{\partial f}{\partial x}(1, \frac{1}{7})(x-1) + \frac{\partial f}{\partial y}(1, \frac{1}{7})(y - \frac{1}{7})$

$= \frac{17}{2} + 8(x-1) + 8(y - \frac{1}{7})$

3 a) $\frac{1+i+(1+i)^2}{3-i} = \frac{1+i+(1+i)(1+i)}{3-i} = \frac{1+i+1+i+i+\overset{-1}{i^2}}{3-i}$

$= \frac{1+3i}{3-i} = \frac{(1+3i)(3+i)}{(3-i)(3+i)} = \frac{3+i+9i+3i^2}{3^2+1^2} = \frac{10i}{10} = \boxed{i}$

b) Schrijf $8+8\sqrt{3}\,i$ in de vorm $r(\cos\varphi + i\sin\varphi)$ met $r>0$

$r = |8+8\sqrt{3}\,i| = \sqrt{8^2+(8\sqrt{3})^2} = \sqrt{64+64\times 3} = \sqrt{256} = 16$

$\cos\varphi = \frac{8}{16} = \frac{1}{2}$, $\sin\varphi = \frac{8\sqrt{3}}{16} = \frac{1}{2}\sqrt{3}$, $\varphi = \frac{1}{3}\pi$.

Dus $8+8\sqrt{3}\,i = 16\left(\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi\right)$

$(8+8\sqrt{3}\,i)^{30} = 16^{30}\left(\cos\frac{30}{3}\pi + i\sin\frac{30}{3}\pi\right) = 2^{120}(\cos 10\pi + i\sin 10\pi)$
$= \boxed{2^{120}}$

c) $z^2+(3+i)z+3i=0$

zoek a,b met $a+b=3+i$, $ab=3i$.

Dan is $z^2+(3+i)z+3i = (z+a)(z+b)$. We kunnen nemen: $a=3, b=i$

Dus $z^2+(3+i)z+3i = (z+3)(z+i)$

$z^2+(3+i)z+3i=0 \iff (z+3)(z+i)=0 \iff \boxed{z=-3,\ z=-i}$

d) $z^{10}+1024i=0 \iff z^{10}=-1024i$

Schrijf $-1024i$ in de vorm $\rho(\cos\psi + i\sin\psi)$ met $\rho>0$

$\rho = |-1024i| = 1024$, $\cos\psi = 0$, $\sin\psi = -1$

$-1024 = 1024\cdot\left(\cos(-\frac{1}{2}\pi) + i\sin(-\frac{1}{2}\pi)\right) = 2^{10}\left(\cos(-\frac{1}{2}\pi) + i\sin(-\frac{1}{2}\pi)\right)$

oplossingen: $\boxed{z_k = 2\left(\cos\left(-\frac{1}{20}\pi + \frac{2k\pi}{10}\right) + i\sin\left(-\frac{1}{20}\pi + \frac{2k\pi}{10}\right)\right) \quad (k=0,1,\ldots,9)}$

Hier is gebruikt: $z^q = \rho(\cos\psi + i\sin\psi)$, $\rho>0$

$\iff z = \sqrt[q]{\rho}\cdot\left(\cos\left(\frac{\psi}{q}+\frac{2k\pi}{q}\right) + i\sin\left(\frac{\psi}{q}+\frac{2k\pi}{q}\right)\right)$, $k=0,1,\ldots,q$

④

3e) Schrijf $-e = r(\cos\varphi + i\sin\varphi)$, met $r>0$

$r = |-e| = e$, $\cos\varphi = -1$, $\sin\varphi = 0$, $-e = e(\cos\pi + i\sin\pi)$

$$e^z = -e \iff z = \ln e + \pi i + 2k\pi i \quad (k\in\mathbb{Z})$$
$$= \boxed{1 + (\pi + 2k\pi)i \quad (k\in\mathbb{Z})}$$

Hier is gebruikt: $e^z = r(\cos\varphi + i\sin\varphi) \iff$
$z = \ln r + (\varphi + 2k\pi)i$, $k\in\mathbb{Z}$.

④ $$\sum_{n=0}^{\infty} \frac{3\times 4^n - 2\times 6^n + 1}{7^n} = 3\sum_{n=0}^{\infty}\left(\frac{4}{7}\right)^n - 2\sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{7}\right)^n$$

$$= \frac{3}{1-\frac{4}{7}} - 2\,\frac{1}{1-\frac{6}{7}} + \frac{1}{1-\frac{1}{7}} = \frac{3}{3/7} - 2\,\frac{1}{1/7} + \frac{1}{6/7}$$

$$= 7 - 14 + \frac{7}{6} = -\frac{42}{6} + \frac{7}{6} = \boxed{\frac{-35}{6}}$$