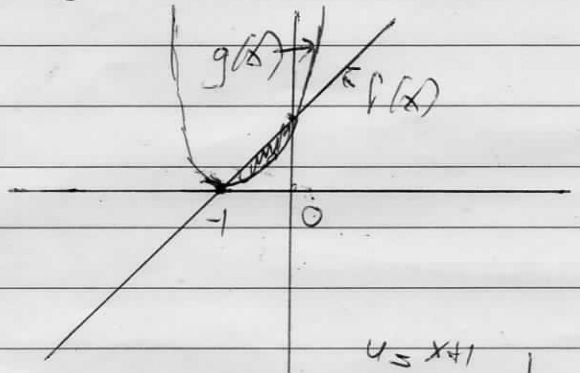
UITWERKING HERKANSING CONTINUE WISKUNDE 2
20/7/2022

(1) a) Snijpunten: $f(x) = g(x) \Leftrightarrow x+1 = (x+1)^4 \Leftrightarrow (x+1)^4 - (x+1) = 0$
$\Leftrightarrow (x+1)((x+1)^3 - 1) = 0 \Leftrightarrow x(x+1)^3 = 0 \Leftrightarrow x = 0$ of $x = -1$

Gebied:



Oppervlakte: $\int_{-1}^{0} (x+1 - (x+1)^4)\,dx \overset{u = x+1}{=} \int_0^1 (u - u^4)\,du$

$= \left[\frac{1}{2}u^2 - \frac{1}{5}u^5\right]_0^1 = \left(\frac{1}{2} - \frac{1}{5}\right) - 0 = \frac{5}{10} - \frac{2}{10} = \boxed{\frac{3}{10}}$

b) $\int \frac{\sin(2x^{3/7})}{x^{4/7}}\,dx = \int \frac{7}{6} \sin u\,du = -\frac{7}{6}\cos u + C = \boxed{-\frac{7}{6}\cos(2x^{3/7}) + C}$

$u = 2x^{3/7}$, $du = \frac{6}{7}x^{-4/7}$
$x^{-4/7}dx = \frac{7}{6}du$

c) $\int x e^{-5x-1}\,dx = x \cdot \left(-\frac{1}{5}e^{-5x-1}\right) - \int -\frac{1}{5}e^{-5x-1} \cdot x'\,dx$

$f(x) = x$
$g'(x) = e^{-5x-1}$
$g(x) = -\frac{1}{5}e^{-5x-1}$

$= -\frac{1}{5}xe^{-5x-1} + \frac{1}{5}\int e^{-5x-1}\,dx$

$= -\frac{1}{5}xe^{-5x-1} - \frac{1}{25}e^{-5x-1} + C$

$\int_0^{\infty} xe^{-5x-1}\,dx = \lim_{B\to\infty} \int_0^B xe^{-5x-1}\,dx = \lim_{B\to\infty} \left[-\frac{1}{5}xe^{-5x-1} - \frac{1}{25}e^{-5x-1}\right]_0^B$

$= \lim_{B\to\infty} -\frac{1}{5}Be^{-5B-1} - \frac{1}{25}e^{-5B-1} - \left(0 - \frac{1}{25}e^{-1}\right) = \boxed{\frac{e^{-1}}{25}}$

($-\frac{1}{5}Be^{-5B-1} \to 0$, $\frac{1}{25}e^{-5B-1} \to 0$)

(2) a) $f(x,y) = x^5 + xy^2 - 5x$

$\frac{\partial f}{\partial x} = 5x^4 + y^2 - 5, \quad \frac{\partial f}{\partial y} = 2xy$

stationaire punten $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$

$\Leftrightarrow 5x^4 + y^2 - 5 = 0, \; 2xy = 0$

$y = 0 \Rightarrow 5x^4 - 5 = 0 \Leftrightarrow x^4 = 1 \Leftrightarrow x = \pm 1 \rightarrow (1,0), (-1,0)$

$x = 0 \Rightarrow y^2 - 5 = 0 \Rightarrow y = \pm\sqrt{5} \rightarrow (0, -\sqrt{5}), (0, \sqrt{5})$

Als we deze punten invullen in $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ zien we dat ze voldoen aan $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$

Dus de vier genoemde punten zijn alle stationaire punten van f.

b) Raakvlak: $z = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1)$

$f(1,1) = 1 + 1 - 5 = -3$

$\frac{\partial f}{\partial x}(1,1) = 5 + 1 - 5 = 1, \quad \frac{\partial f}{\partial y} = 2 \cdot 1 \cdot 1 = 2$

Vergelijking raakvlak: $\boxed{z = -3 + 1 \cdot (x-1) + 2 \cdot (y-1)}$

c) $A = \frac{\partial^2 f}{\partial x^2} = 20x^3, \quad B = \frac{\partial^2 f}{\partial y \partial x} = 2y, \quad C = \frac{\partial^2 f}{\partial y^2} = 2x, \quad H = AC - B^2$

	A	B	C	H	
(1, 0)	20	0	2	40 > 0	minimum
(-1, 0)	-20	0	-2	40 > 0	maximum
$(0, \sqrt{5})$	0	$2\sqrt{5}$	0	-20 < 0	zadelpunt
$(0, -\sqrt{5})$	0	$-2\sqrt{5}$	0	-20 < 0	zadelpunt

$\lim_{x\to\infty} f(x,0) = \lim_{x\to\infty} x^5 - 5x = \infty, \quad \lim_{x\to-\infty} f(x,0) = -\infty$

Dus het minimum en maximum zijn $\boxed{\text{relatief}}$

(3) a) Deel eerst de vergelijking door $-i$, dus vermenigvuldig met $\frac{1}{-i} = \frac{i}{-i.i} = \frac{i}{1} = i$.

$-i z^2 + (1-4i) z + 4 = 0 \Leftrightarrow z^2 + i(1-4i) z + 4i = 0 \Leftrightarrow$

$z^2 + (i - 4i^2) z + 4i = 0 \Leftrightarrow z^2 + (4+i) z + 4i = 0$

Zoek u, v zodat $u+v = 4+i$, $u.v = 4i$. Neem $u = 4$, $v = i$

Dus $z^2 + (4+i) z + 4i = 0 \Leftrightarrow (z+4)(z+i) = 0 \Leftrightarrow$ $z = -4$ of $z = -i$

b) Schrijf eerst $1 - \sqrt{3} i$ in de vorm $r(\cos\varphi + i \sin\varphi)$

$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\cos\varphi = \frac{1}{2}$, $\sin\varphi = -\frac{1}{2}\sqrt{3} \Rightarrow \varphi = -\frac{1}{3}\pi$

$1 - \sqrt{3} i = 2(\cos(-\frac{1}{3}\pi) + i \sin(-\frac{1}{3}\pi))$

$(1-\sqrt{3} i)^{100} = 2^{100}(\cos(-\frac{100}{3}\pi) + i \sin(-\frac{100}{3}\pi))$

$= 2^{100}(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi) = 2^{100}(-\frac{1}{2} + \frac{1}{2} i\sqrt{3})$

$-\frac{100}{3}\pi = -34\pi + \frac{2}{3}\pi$

$\frac{(1-\sqrt{3} i)^{100}}{1+i} = \frac{2^{100}(-\frac{1}{2} + \frac{1}{2} i\sqrt{3})(1-i)}{(1+i)(1-i)} = 2^{99}(-\frac{1}{2} + \frac{1}{2} i + \frac{1}{2} i\sqrt{3} - \frac{1}{2}\sqrt{3}.i^2)$

$= 2^{99}(-\frac{1}{2} + \frac{1}{2}\sqrt{3} + (\frac{1}{2} + \frac{1}{2}\sqrt{3}) i)$

c) Stel $u = 10z + 3$. We lossen eerst $e^u = \frac{e^5 + e^5\sqrt{3} i}{2}$ op

Schrijf $\frac{e^5 + e^5\sqrt{3} i}{2} = r(\cos\varphi + i \sin\varphi)$

$r = \sqrt{(e^{10} + e^{10} \cdot 3)/4} = \sqrt{e^{10}} = e^5$, $\cos\varphi = \frac{\frac{1}{2}e^5}{e^5} = \frac{1}{2}$, $\sin\varphi = \frac{\frac{1}{2}\sqrt{3}e^5}{e^5} = \frac{1}{2}\sqrt{3}$

$\rightarrow \varphi = \frac{1}{3}\pi$. Dus $\frac{1}{2}(e^5 + e^5\sqrt{3} i) = e^5(\cos \frac{1}{3}\pi + i \sin \frac{1}{3}\pi)$.

Dit geeft als oplossingen $u = \ln e^5 + (\frac{1}{3}\pi + 2k\pi) i$

$= 5 + (\frac{1}{3}\pi + 2k\pi) i \quad (k \in \mathbb{Z})$

Nu is $u = 10z + 3$, dus $z = \frac{u-3}{10} = \frac{1}{5} + (\frac{1}{30} + \frac{1}{5}k\pi) i \quad (k \in \mathbb{Z})$

3d) Schrijf $5^{20}(1-i) = r(\cos\varphi + i\sin\varphi)$

$r = \sqrt{(5^{20})^2 + (5^{20})^2} = \sqrt{5^{40} \cdot 2} = 5^{20}\sqrt{2}$

$\cos\varphi = \frac{5^{20}}{5^{20}\sqrt{2}} = \frac{1}{2}\sqrt{2}$, $\sin\varphi = \frac{-5^{20}}{5^{20}\sqrt{2}} = -\frac{1}{2}\sqrt{2} \rightarrow \varphi = -\frac{1}{4}\pi$

$5^{20}(1-i) = 5^{20}\sqrt{2}(\cos(-\frac{1}{4}\pi) + i\sin(-\frac{1}{4}\pi))$

oplossingen $z_k = \sqrt[20]{5^{20}\sqrt{2}} \cdot \cos(-\frac{1}{80}\pi + \frac{2k\pi}{80}) + i\sin(-\frac{1}{80}\pi + \frac{2k\pi}{20})$

$= 5\sqrt[40]{2}(\cos(-\frac{1}{80}\pi + \frac{k\pi}{10}) + i\sin(-\frac{1}{80}\pi + \frac{k\pi}{10}))$

$(k=0,\dots,19)$

4a) $\sum_{n=2}^{\infty} \frac{4^n + (-3)^n}{5^n} = \sum_{n=2}^{\infty} \left(\frac{4}{5}\right)^n + \sum_{n=2}^{\infty} \left(\frac{-3}{5}\right)^n$

$= \frac{(4/5)^2}{1-\frac{4}{5}} + \frac{(-3/5)^2}{1+\frac{3}{5}} = \frac{16}{25} \cdot 5 + \frac{9}{25} \cdot \frac{5}{8} = \frac{16}{5} + \frac{9}{40} = \frac{8 \cdot 16 + 9}{40} = \frac{137}{40}$

4b) n^2+2 heeft orde van grootte n^2,

$n^{31/10}-1$ heeft orde van grootte $n^{31/10}$

Dus $a_n := \frac{n^2+2}{n^{\frac{31}{10}}-1}$ heeft orde van grootte $\frac{n^2}{n^{\frac{31}{10}}} = n^{-\frac{11}{10}}$

Vergelijk a_n met $b_n = n^{-11/10}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(n^2+2) n^{11/10}}{n^{\frac{31}{10}}-1} = \lim_{n\to\infty} \frac{n^{\frac{31}{10}} + 2n^{\frac{11}{10}}}{n^{\frac{31}{10}}-1}$

$= \lim_{n\to\infty} \frac{1+2n^{-2}}{1-n^{-31/10}} = 1 < \infty$

De reeks $\sum_{n=1}^{\infty} n^{-11/10}$ is convergent.

Volgens het vergelijkingskenmerk is $\sum_{n=2}^{\infty} \frac{n^2+2}{n^{\frac{31}{10}}-1}$ convergent