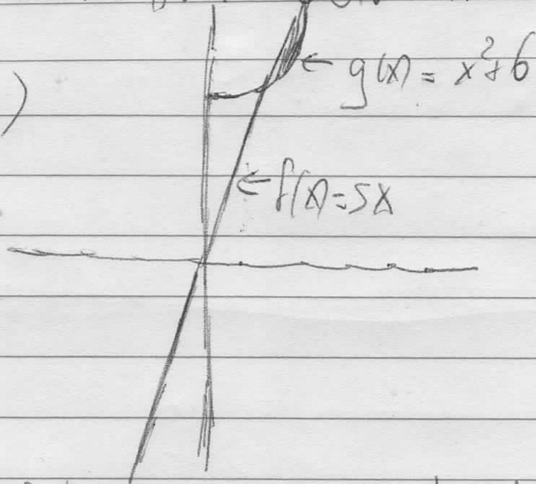
UITWERKING CONTINUE WISKUNDE 2, 27/3/2019

1 a)



Snijpunten van de grafieken van $f(x)$ en $g(x)$:

$5x = x^2+6 \Leftrightarrow x^2-5x+6=0$

$\Leftrightarrow x=2,\ x=3$

Oppervlakte van het gebied $\int_2^3 (f(x)-g(x))\,dx = \int_2^3 (5x-x^2-6)\,dx$

$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 - 6x\right]_2^3 = \left(\frac{5}{2}\times 9 - \frac{1}{3}\times 27 - 6\times 3\right) - \left(\frac{5}{2}\times 4 - \frac{1}{3}\times 8 - 6\times 2\right)$

$= \left(\frac{45}{2} - 9 - 18\right) - \left(10 - \frac{8}{3} - 12\right) = -4\frac{1}{2} - \left(-\frac{14}{3}\right) = 4\frac{2}{3} - 4\frac{1}{2} = \boxed{\frac{1}{6}}$

b) $\int (x^2+x+1)\ln x \, dx = \ln x \cdot \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right) - \int \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right)\cdot\frac{1}{x}\,dx$

$f(x) = \ln x$
$g'(x) = x^2+x+1$
$g(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$

$= \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right)\ln x - \int \left(\frac{1}{3}x^2 + \frac{1}{2}x + 1\right)dx$

$= \boxed{\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right)\ln x - \frac{1}{9}x^3 - \frac{1}{4}x^2 - x + C}$

c) $\int \frac{\sin x^{1/3}}{x^{2/3}}\,dx = \int \sin u \cdot 3\,du = -3\cos u + C$

$= -3\cos x^{1/3} + C$

$u = x^{1/3}$
$du = \frac{1}{3}x^{-2/3}\,dx$
$x^{-2/3}\,dx = 3\,du$

$\int_0^{(\pi/2)^3} \frac{\sin x^{1/3}}{x^{2/3}}\,dx = \lim_{t\to 0}\int_t^{(\pi/2)^3} (\ldots) = \lim_{t\to 0}\left[-3\cos x^{1/3}\right]_t^{(\pi/2)^3}$

$= -3\cos\frac{\pi}{2} + 3\cdot\cos 0 = \boxed{3}$

2

(2) a) $(x+2y)^2+(x^2-1)^2-1 = x^2+4xy+4y^2+x^4-2x^2+1-1$
$= x^4-x^2+4xy+4y^2 = f(x,y)$

b) $\frac{\partial f}{\partial x} = 4x^3-2x+4y$, $\frac{\partial f}{\partial y} = 4x+8y$

Uit $\frac{\partial f}{\partial y}=0$ volgt: $y=-\frac{1}{2}x$. Invullen in $\frac{\partial f}{\partial x}=0$ geeft

$4x^3-2x-2x = 4x^3-4x = 4x(x^2-1)=0$, dus $x=0$, $x=1$, of $x=-1$
Dit geeft de stationaire punten: $(0,0)$, $(1,-\frac{1}{2})$, $(-1,\frac{1}{2})$

c) $A = \frac{\partial^2 f}{\partial x^2} = 12x^2-2$, $B = \frac{\partial^2 f}{\partial y \partial x} = 4$, $C = \frac{\partial^2 f}{\partial y^2} = 8$, $H = AC-B^2$

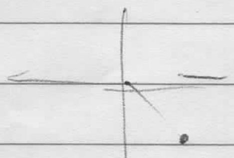
	A	B	C	H	
$(0,0)$	-2	4	8	$-32<0$	zadelpunt
$(1,-\frac{1}{2})$	$10>0$	4	8	$64>0$	minimum
$(-1,\frac{1}{2})$	$10>0$	4	8	$64>0$	minimum

<u>Opmerking</u> $f(1,-\frac{1}{2}) = f(-1,\frac{1}{2}) = -1$ (volgt uit a)
$f(x,y) \geq -1$ voor alle x,y (volgt uit a))
Dus beide minima zijn <u>absoluut</u>

d) <u>raakvlak</u>: $z = f(2,2)+\frac{\partial f}{\partial x}(2,2)(x-2)+\frac{\partial f}{\partial y}(2,2)(y-2)$

$= 44+36(x-2)+24(y-2)$

(3) a) $|z| = \sqrt{3^2+4^2} = \sqrt{25} = 5$, $|w| = \sqrt{24^2+7^2} = \sqrt{576+49} = \sqrt{625} = 25$,
$|z^4/w^2| = 5^4/25^2 = \boxed{1}$



b) Schrijf $5-5i = r(\cos\varphi + i\sin\varphi)$.
$r = |5-5i| = \sqrt{5^2+(-5)^2} = \sqrt{50}$, $\cos\varphi = \frac{5}{\sqrt{50}} = \frac{1}{2}\sqrt{2}$, $\sin\varphi = \frac{-5}{\sqrt{50}} = -\frac{1}{2}\sqrt{2}$

dus $5-5i = \sqrt{50}\,(\cos(-\frac{1}{4}\pi)) + i\sin(-\frac{1}{4}\pi))$

(3)

Bijgevolg is $(5-5i)^{20} = \sqrt{50}^{20}\left(\cos\left(\frac{-20}{4}\pi\right) + i\sin\left(\frac{-20}{4}\pi\right)\right)$

$= 50^{10}(\cos\pi + i\sin\pi) = \boxed{-50^{10}}$

c) $z^2 - 4z + 8 = 0$, $D = 4^2 - 4\times 8 = -16 < 0$

oplossingen: $z_{1,2} = \frac{4 \pm i\sqrt{16}}{2} = \boxed{2 \pm 2i}$

d) $z^6 - 4z^3 + 8 = 0$. Stel $w = z^3$. Dan $w^2 - 4w + 8 = 0$,

dus $w = 2+2i$ of $w = 2-2i$

$|2+2i| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$,

$2+2i = r(\cos\varphi + i\sin\varphi)$, $\cos\varphi = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$, $\sin\varphi = \frac{1}{2}\sqrt{2}$

$\Rightarrow 2+2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$|2-2i| = 2\sqrt{2}$

$2-2i = r(\cos\varphi + i\sin\varphi)$, $\cos\varphi = \frac{1}{\sqrt{2}}$, $\sin\varphi = -\frac{1}{\sqrt{2}}$

$\Rightarrow 2-2i = 2\sqrt{2}\left(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}\right)$

oplossingen van $z^3 = 2+2i$

$z_{0,1,2} = \sqrt[3]{2\sqrt{2}}\left(\cos\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right)$

$= \boxed{\sqrt{2}\left(\cos\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right) \quad k = 0,1,2}$

oplossingen van $z^3 = 2-2i$

$z'_{0,1,2} = \sqrt[3]{2\sqrt{2}}\left(\cos\left(-\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(-\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right)$

$= \boxed{\sqrt{2}\left(\cos\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right) \quad k = 0,1,2}$

e) Oplossing van $e^z = q$, $q = r(\cos\varphi + i\sin\varphi)$:

$z = \ln r + \varphi i + 2k\pi i \quad (k \in \mathbb{Z})$

Hier: $q = -1$, $r = 1$, $\varphi = \pi$. Oplossing: $\boxed{z = \pi i + 2k\pi i \ (k \in \mathbb{Z})}$

4 $\sum_{n=0}^{\infty} \frac{7(-2)^n + 2\times 3^n + 8\times 4^n}{5^n} = \sum_{n=0}^{\infty}\left(7\left(\frac{-2}{5}\right)^n + 2\cdot\left(\frac{3}{5}\right)^n + 8\left(\frac{4}{5}\right)^n\right)$

$= 7\sum_{n=0}^{\infty}\left(\frac{-2}{5}\right)^n + 2\sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n + 8\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n$

$= \frac{7}{1-(-2/5)} + \frac{2}{1-(3/5)} + \frac{8}{1-(4/5)} = \frac{7}{7/5} + \frac{2}{2/5} + \frac{8}{1/5}$

$= 5 + 5 + 40 = \boxed{50}$