UITWERKING TENTAMEN OW2 (VERSIE 2), 12-6-2020

1 a) We bepalen eerst de snijpunten van de grafieken van f en g.

Er geldt: $f(x)=g(x) \Leftrightarrow 4x = x^2-5 \Leftrightarrow x^2-4x-5=0$

$\Leftrightarrow (x+1)(x-5)=0 \Leftrightarrow x=-1,\ x=5$

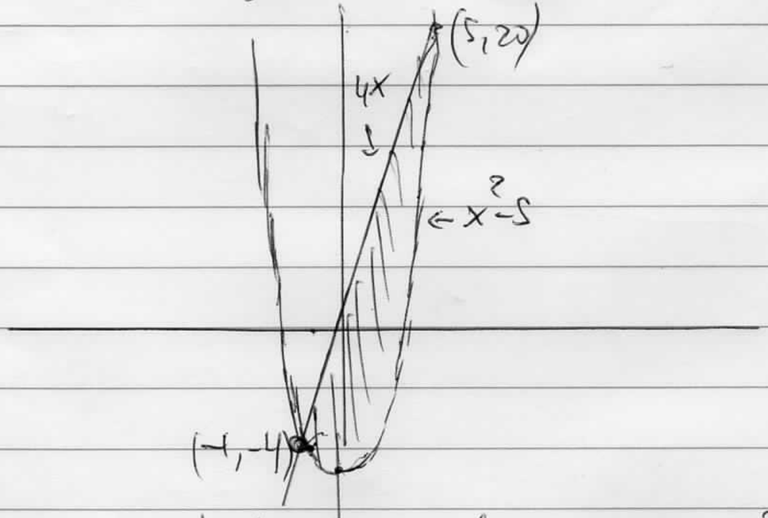
De snijpunten zijn $(-1,-4),(5,20)$.

$$\text{oppervlakte} = \int_{-1}^{5}(f(x)-g(x))\,dx = \int_{-1}^{5}(4x-x^2+5)\,dx$$

$$= \left[2x^2-\tfrac{1}{3}x^3+5x\right]_{-1}^{5} = \left(50-\tfrac{125}{3}+25\right)-\left(2-\left(-\tfrac{1}{3}\right)+(-5)\right)$$

$$= \tfrac{100}{3}-\left(-\tfrac{8}{3}\right) = \tfrac{108}{3} = \boxed{36}$$

B)



b) Gebruik de substitutieregel met $u=x^3+1$, $du=3x^2dx$, $x^2dx=\frac{1}{3}du$

$$\int \sqrt[4]{x^3+1}\cdot x^2dx = \int u^{1/4}\cdot\tfrac{1}{3}\,du = \tfrac{1}{3}\cdot\tfrac{4}{5}u^{5/4}+C = \tfrac{4}{15}u^{5/4}+C = \tfrac{4}{15}(x^3+1)^{5/4}+C$$

c) Gebruik partiële integratie

$$\int(2x+5)e^{-4x}dx = (2x+5)\left(-\tfrac{1}{4}e^{-4x}\right)-\int\left(-\tfrac{1}{4}e^{-4x}\right)\cdot 2\,dx$$

$f(x)=2x+5,\ g'(x)=e^{-4x},\ g(x)=-\frac{1}{4}e^{-4x}$

$$= -\tfrac{1}{4}(2x+5)e^{-4x}+\tfrac{1}{2}\int e^{-4x}dx = -\tfrac{1}{4}(2x+5)e^{-4x}-\tfrac{1}{8}e^{-4x}+C$$

$$= \left(-\tfrac{1}{2}x-\tfrac{11}{8}\right)e^{-4x}+C$$

Dus $\int_0^{\infty}(2x+5)e^{-4x}dx = \lim_{B\to\infty}\int_0^{B}(2x+5)e^{-4x}dx = \lim_{B\to\infty}\left(\left(-\frac{1}{2}x-\frac{11}{8}\right)e^{-4x}\right]_0^B$

$= \lim_{B\to\infty}\left(\left(-\frac{1}{2}B-\frac{11}{8}\right)e^{-4B}-\left(-\frac{11}{8}\right)\right) = \boxed{\frac{11}{8}}$ ($Be^{-4B}, e^{-4B} \to 0$ als $B\to\infty$)

2 a) $\frac{\partial f}{\partial x} = 2(x^2-4)\cancel{2x}2x+2(x+y) = 4x(x^2-4)+2(x+y)$,

$\frac{\partial f}{\partial y} = 2(x+y)$

stationaire punten: $\frac{\partial f}{\partial x}=0,\ \frac{\partial f}{\partial y}=0 \Leftrightarrow \left\{\begin{array}{l}4x(x^2-4)+2(x+y)=0\\ 2(x+y)=0\end{array}\right\}$

$\Leftrightarrow \left\{\begin{array}{l}4x(x^2-4)=0\\ y=-x\end{array}\right\} \Leftrightarrow (x,y)=(0,0),(2,-2),(-2,2)$

b) $\frac{\partial^2 f}{\partial x^2} = 4x\cdot 2x + 4(x^2-4)\cancel{+2} = 12x^2-16 = A$,

$\frac{\partial^2 f}{\partial y\partial x} = \frac{\partial^2 f}{\partial x\partial y} = 2 = B,\ \frac{\partial^2 f}{\partial y^2} = 2 = C,\quad H = AC-B^2$

	A	B	C	H	
(0,0)	-16	2	2	-36<0	zadelpunt
(2,-2)	32>0	2	2	60>0	minimum
(-2,2)	32>0	2	2	60>0	minimum.

Er geldt $f(2,-2)=f(-2,2)=0$, $f(x,y)\geq 0$ voor alle x,y
(som van twee kwadraten)
Dus de minima zijn <u>absoluut</u>

c) Vergelijking raakvlak $z = f(1,2) + \frac{\partial f}{\partial x}(1,2)(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2)$

$f(1,2)=18$, $\frac{\partial f}{\partial x}(1,2) = -6$, $\frac{\partial f}{\partial y}(1,2)=6$

$\boxed{z = 18-6(x-1)+6(y-2)}$

3 a) $|z| = \sqrt{9^2+40^2} = \sqrt{1681} = 41$, $|w| = \sqrt{9^2+(-40)^2} = 41$,

$$\left|\frac{z^{1000}}{w^{1001}}\right| = \frac{|z|^{1000}}{|w|^{1001}} = \frac{41^{1000}}{41^{1001}} = \boxed{\frac{1}{41}}$$

b) We berekenen eerst $(\sqrt{3}+i)^{51}$. Schrijf $\sqrt{3}+i$ in de vorm $r(\cos\varphi + i\sin\varphi)$. Er geldt $r = |\sqrt{3}+i| = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$, $\cos\varphi = \frac{1}{2}\sqrt{3}$, $\sin\varphi = \frac{1}{2}$, dus we kunnen $\varphi = \frac{1}{6}\pi$ nemen.
Dit geeft $(\sqrt{3}+i)^{51} = 2^{51}(\cos\frac{51}{6}\pi + i\sin\frac{51}{6}\pi) = 2^{51}(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi)$
$= 2^{51}\cdot i$

Dit geeft $\left(\frac{3+i}{(\sqrt{3}+i)^{51}}\right) = \frac{3+i}{2^{51}i} = 2^{-51}\cdot -i(3+i) = 2^{-51}(-i^2-3i)$
$= \boxed{2^{-51}(1-3i)}$

c) Schrijf $1024-1024i$ in de vorm $r(\cos\varphi + i\sin\varphi)$
$r = \sqrt{1024^2+(-1024)^2} = 1024\sqrt{1^2+(-1)^2} = 1024\sqrt{2}$
$\cos\varphi = \frac{1}{\sqrt{2}}$, $\sin\varphi = -\frac{1}{\sqrt{2}}$, dus we kunnen $\varphi = -\frac{1}{4}\pi$ nemen

Dan zijn de oplossingen

$$z_k = \sqrt[5]{r}\left(\cos\left(\frac{\varphi}{5}+\frac{2k\pi}{5}\right) + i\sin\left(\frac{\varphi}{5}+\frac{2k\pi}{5}\right)\right)$$
$$= \sqrt[5]{1024\sqrt{2}}\cdot\left(\cos\left(-\frac{1}{20}\pi+\frac{2k\pi}{5}\right) + i\sin\left(-\frac{1}{20}\pi+\frac{2k\pi}{5}\right)\right) \quad k=0,1,2,3,4$$
$$= 2^{21/10}\left(\cos\left(-\frac{1}{20}\pi+\frac{2k\pi}{5}\right) + i\sin\left(-\frac{1}{20}\pi+\frac{2k\pi}{5}\right)\right) \quad k=0,1,2,3,4$$

d) Schrijf $w = e^z$. Dit geeft $w^2+8w+15=0$ dwz $(w+3)(w+5)=0$ dwz $w=-3$ of $w=-5$.
Er geldt: $-3 = 3(\cos\pi + i\sin\pi)$, $-5 = 5(\cos\pi + i\sin\pi)$
Dus de oplossingen zijn:

$z_k = \ln 3 + (\pi + 2k\pi)i \quad (k\in\mathbb{Z})$; $\quad z_k' = \ln 5 + (\pi+2k\pi)i \quad (k\in\mathbb{Z})$.

4 a) $\sum_{n=0}^{\infty}\left(\frac{2^n}{3^n}-\frac{4^n}{5^n}\right)=\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n-\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n$

$= \frac{1}{1-\frac{2}{3}}-\frac{1}{1-\frac{4}{5}} = 3-5=\boxed{-2}$

b) We passen het quotiëntkenmerk toe met $a_n = \frac{1}{n!\cdot n^3}$

Dan is $a_{n+1} = \frac{1}{(n+1)!\,(n+1)^3}$

Dus $\frac{a_{n+1}}{a_n} = \frac{1}{(n+1)!\,(n+1)^3}\cdot n!\,n^3 = \frac{n^3}{(n+1)^3}\cdot\frac{n!}{(n+1)!}$

$= \left(\frac{n}{n+1}\right)^3\cdot\frac{1\cdot 2\cdots n}{1\cdot 2\cdots n(n+1)} = \left(\frac{n}{n+1}\right)^3\cdot\frac{1}{n+1}$,

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^3\cdot\frac{1}{n+1} = 0<1$

Dus $\sum_{n=1}^{\infty}\frac{1}{n!\,n^3}$ is convergent.