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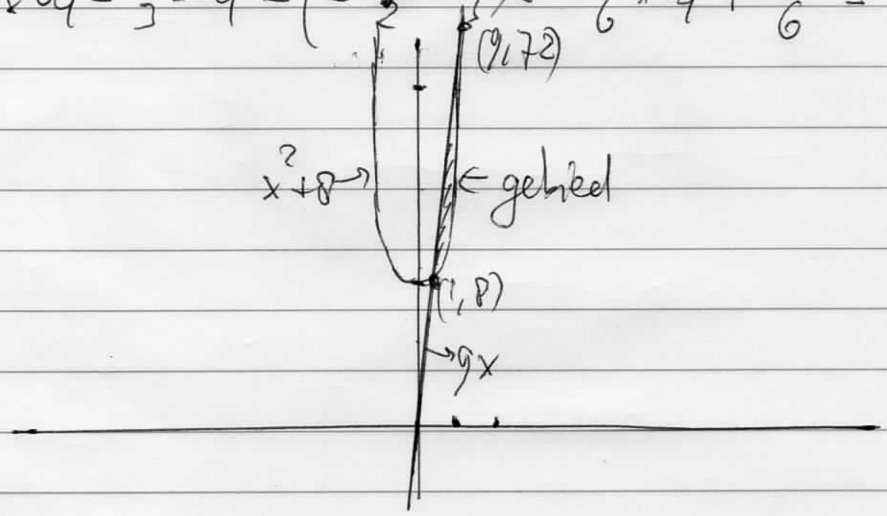
UITWERKING TENTAMEN OW2 VERSIE 3 12-6-2020

1a) We bepalen eerst de snijpunten van de grafieken van $f(x)$ en $g(x)$. Er geldt: $f(x)=g(x) \Leftrightarrow 9x = x^2+8 \Leftrightarrow x^2-9x+8=0$ $\Leftrightarrow (x-1)(x-8)=0$. Dus de snijpunten zijn $(1,9)$, $(8,72)$.

~~Inhoud~~ Oppervlakte: $\int_1^8 (f(x)-g(x))\,dx = \int_1^8 (9x-x^2-8)\,dx$

$$= \left[\frac{9}{2}x^2 - \frac{1}{3}x^3 - 8x\right]_1^8 = \left(\frac{9}{2}\times 64 - \frac{1}{3}\times 512 - 64\right) - \left(\frac{9}{2} - \frac{1}{3} - 8\right)$$

$$= \frac{7}{2}\times 64 - \frac{8}{3}\times 64 - \left(-\frac{7}{2} - \frac{1}{3}\right) = \frac{5}{6}\times 64 + \frac{23}{6} = \frac{343}{6} = \boxed{57\tfrac{1}{6}}$$



b) Pas de substitutieregel toe met $u = x^2+1$, $du = 2x\,dx$, $x\,dx = \frac{1}{2}du$

$$\int \sqrt[3]{x^2+1}\,dx = \int u^{1/3}\cdot\frac{1}{2}du = \frac{1}{2}\cdot\frac{3}{4}\cdot u^{4/3} + C = \frac{3}{8}(x^2+1)^{4/3} + C$$

c) Pas partiële integratie toe met $f(x) = 6x+7$, $g'(x) = e^{-5x}$, $g(x) = -\frac{1}{5}e^{-5x}$. Dit geeft

$$\int (6x+7)e^{-5x}\,dx = (6x+7)\cdot\left(-\frac{1}{5}e^{-5x}\right) - \int\left(-\frac{1}{5}e^{-5x}\right)\cdot 6\,dx$$

$$= -\frac{1}{5}(6x+7)e^{-5x} + \frac{6}{5}\int e^{-5x}\,dx = -\frac{1}{5}(6x+7)e^{-5x} - \frac{6}{25}e^{-5x} + C$$

$$= \left(-\frac{6}{5}x - \frac{41}{25}\right)e^{-5x} + C$$

1 c) (vervolg) $\int_0^\infty (6x+7)e^{-5x}\,dx = \lim_{B\to\infty} \int_0^B (6x+7)e^{-5x}\,dx$

$= \lim_{B\to\infty} \left[\left(-\frac{6}{5}x - \frac{41}{25}\right)e^{-5x}\right]_0^B = \lim_{B\to\infty}\left(\left(-\frac{6}{5}B - \frac{41}{25}\right)e^{-5B} - \left(-\frac{41}{25}\right)\right) = \boxed{\frac{41}{25}}$

$(Be^{-5B},\ e^{-5B} \to 0$ als $B\to\infty)$

2 a) $\frac{\partial f}{\partial x} = 2(x-y) + \cancel{\ldots} = 2(x-y)\ \cancel{\ldots}$,

$\frac{\partial f}{\partial y} = -2(x-y) + 2(y^2-9)\cdot 2y = -2(x-y) + 4y(y^2-9)$

stationaire punten: $\frac{\partial f}{\partial x} = 0,\ \frac{\partial f}{\partial y} = 0 \Leftrightarrow \begin{cases} 2(x-y) = 0 \\ -2(x-y) + 4y(y^2-9) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x-y=0 \\ y(y^2-9)=0 \end{cases} \Leftrightarrow \begin{cases} x=y \\ y=0,3,-3 \end{cases} \Leftrightarrow (x,y) = (0,0), (3,3), (-3,-3)$

b) $\frac{\partial^2 f}{\partial x^2} = 2 = A,\ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -2 = B,$

$\frac{\partial^2 f}{\partial y^2} = 2 + 4y\cdot 2y + 4(y^2-9) = 12y^2 - 34 = C,\quad H = AC - B^2$

	A	B	C	H	
(0,0)	2	-2	-34	$-72<0$	zadelpunt
(3,3)	$2>0$	-2	72	$140>0$	minimum
(-3,-3)	$2>0$	-2	72	$140>0$	minimum

Er geldt $f(3,3) = f(-3,-3) = 0$, $f(x,y) \geq 0$ voor alle (x,y) (namelijk som van twee kwadraten). Dus de minima zijn absoluut.

c) vergelijking raakvlak $z = f(1,2) + \frac{\partial f}{\partial x}(1,2)(x-1) + \frac{\partial f}{\partial y}(1,2)(y-2)$

$f(1,2) = 26,\ \frac{\partial f}{\partial x}(1,2) = -2,\ \frac{\partial f}{\partial y}(1,2) = -38$

$\boxed{z = 26 - 2(x-1) - 38(y-2)}$

3 a) $|z| = \sqrt{4^2+(-1)^2} = \sqrt{17}$, $|w| = \sqrt{4^2+1^2} = \sqrt{17}$

$$|z^{20}/w^{22}| = \frac{|z|^{20}}{|w|^{22}} = \frac{(\sqrt{17})^{20}}{(\sqrt{17})^{22}} = \frac{17^{10}}{17^{11}} = \boxed{\frac{1}{17}}$$

b) We berekenen eerst $(2-2i)^{110}$.
Schrijf $2-2i = r(\cos\varphi + i\sin\varphi)$. $r = |2-2i| = \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$
$\cos\varphi = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$, $\sin\varphi = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$, we kunnen $\varphi = -\frac{1}{4}\pi$ nemen.

$$(2-2i)^{110} = (\sqrt{8})^{110}\left(\cos\left(-\frac{110}{4}\pi\right) + i\sin\left(-\frac{110}{4}\pi\right)\right) = 8^{55}\left(\cos -\tfrac{1}{2}\pi + i\sin -\tfrac{1}{2}\pi\right)$$
$$= -8^{55}i.$$

$$\frac{3-i}{(2-2i)^{110}} = \frac{3-i}{-8^{55}i} = 8^{-55}i(3-i) = 8^{-55}(3i-i^2) = \boxed{8^{-55}(1+3i)}$$

c) Schrijf $512\left(\frac{1}{2}-\frac{1}{2}\sqrt{3}i\right) = r(\cos\varphi + i\sin\varphi)$
$r = |512(\frac{1}{2}-\frac{1}{2}\sqrt{3}i)| = \sqrt{512^2\left(\frac{1}{4}+\frac{3}{4}\right)} = 512$, $\cos\varphi = \frac{1}{2}$, $\sin\varphi = -\frac{1}{2}\sqrt{3}$
dus we kunnen $\varphi = -\frac{1}{3}\pi$ nemen

Oplossing $z_k = \sqrt[9]{512}\left(\cos\left(\frac{\varphi}{9}+\frac{2k\pi}{9}\right) + i\sin\left(\frac{\varphi}{9}+\frac{2k\pi}{9}\right)\right)$

$$= 2\left(\cos\left(\frac{-1}{27}\pi + \frac{2k\pi}{9}\right) + i\sin\left(\frac{-1}{27}\pi + \frac{2k\pi}{9}\right)\right) \quad k=0,1,2,3,4,5,6,7,8$$

d) Stel $e^{z} = w$. Dan is $w^2+5w+4=0$, dus $(w+1)(w+4)=0$,
dus $w=-1$, $w=-4$. Er geldt: $-1 = \cos\pi + i\sin\pi$, $-4 = 4(\cos\pi + i\sin\pi)$
Dus de oplossingen zijn:

$z_k = (\pi+2k\pi)i \quad (k\in\mathbb{Z})$; $z_k' = \ln 4 + (\pi+2k\pi)i \quad (k\in\mathbb{Z})$

4 a) $$\sum_{n=0}^{\infty}\left(\frac{10^n}{11^n}-\frac{20^n}{21^n}\right)=\sum_{n=0}^{\infty}\left(\frac{10}{11}\right)^n-\sum_{n=0}^{\infty}\left(\frac{20}{21}\right)^n$$

$$=\frac{1}{1-\frac{10}{11}}-\frac{1}{1-\frac{20}{21}}=11-21=\boxed{-10}$$

b) We passen het quotiëntkenmerk toe met $a_n=\frac{n^{12}}{n!}$.
Dan is
$$a_{n+1}=\frac{(n+1)^{12}}{(n+1)!},$$
dus $$\frac{a_{n+1}}{a_n}=\frac{(n+1)^{12}}{(n+1)!}\Big/\frac{n^{12}}{n!}=\frac{(n+1)^{12}}{(n+1)!}\cdot\frac{n!}{n^{12}}=\left(\frac{n+1}{n}\right)^{12}\cdot\frac{1\cdot 2\cdots n}{1\cdot 2\cdots(n+1)}$$

$$=\left(1+\frac{1}{n}\right)^{12}\frac{1}{n+1}.$$ Dus $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=0.$

Dus de reeks convergeert.