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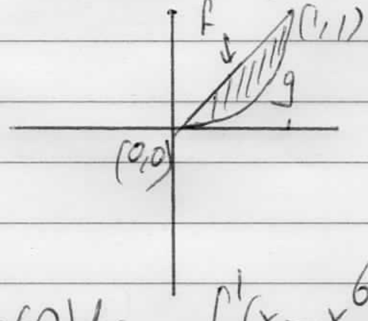
UITWERKING TENTAMEN CONTINUE WISKUNDE 2
31/5/2021

1 a) Snijpunten: $f(x)=g(x) \Leftrightarrow x^6 = x \Leftrightarrow x^6 - x = 0 \Leftrightarrow x(x^5-1)=0$
$\Leftrightarrow x=0, x=1$. Snijpunten: $(0,0), (1,1)$



oppervlakte: $\int_0^1 (f(x)-g(x))dx = \int_0^1 (x-x^6)dx = \left[\frac{1}{2}x^2 - \frac{1}{7}x^7\right]_0^1$
$= \frac{1}{2} - \frac{1}{7} - (0-0) = \frac{7}{14} - \frac{2}{14} = \boxed{\frac{5}{14}}$

b) Substitutie $u=e^x$

$$\int \frac{e^x}{\sqrt{e^x-1}}dx \underset{\substack{u=e^x \\ du=e^x dx}}{=} \int \frac{du}{\sqrt{u-1}} = 2(u-1)^{1/2}+C = 2(e^x-1)^{\frac{1}{2}}+C$$

$$\int_0^{100} \frac{e^x dx}{\sqrt{e^x-1}} = \lim_{t \downarrow 0} \int_t^{100} \frac{e^x dx}{\sqrt{e^x-1}} = \lim_{t\downarrow 0} \left[2(e^x-1)^{1/2}\right]_t^{100}$$

(want $e^x-1=0$ voor $x=0$)

$$= 2(e^{100}-1)^{1/2} - \lim_{t\downarrow 0} \underbrace{2(e^t-1)^{1/2}}_{=0} = \boxed{2(e^{100}-1)^{1/2}}$$

c) $\int (3x+5)\cos 4x\, dx \underset{\uparrow}{=} (3x+5)\cdot\frac{1}{4}\sin 4x - \int \frac{1}{4}\sin 4x \cdot \overset{3}{(3x+5)'}dx$

$f(x)=3x+5$
$g'(x)=\cos 4x$
$g(x)=\frac{1}{4}\sin 4x$

$= \left(\frac{3}{4}x+\frac{5}{4}\right)\sin 4x - \int \frac{3}{4}\sin 4x\, dx$

$= \boxed{\left(\frac{3}{4}x+\frac{5}{4}\right)\sin 4x + \frac{3}{16}\cos 4x + C}$

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2 a) $(x^4-y^4)^2+2(x^2y^2-1)^2+4(xy-1)^2-6$

$= x^8-2x^4y^4+y^4+2(x^4y^4-2x^2y^2+1)+4(x^2y^2-2xy+1)-6$

$= x^8-2x^4y^4+2x^4y^4+y^4-4x^2y^2+2+4x^2y^2-8xy+4-6$

$= x^8-8xy+y^8.$

b) $\frac{\partial f}{\partial x}=8x^7-8y, \quad \frac{\partial f}{\partial x}=-8x+8y^7$

stationaire punten: $\frac{\partial f}{\partial x}=0, \frac{\partial f}{\partial y}=0 \Longleftrightarrow$

$8x^7=8y, \ 8x=8y^7$. ~~Dit geeft 8^7y^{49}~~ $\Longleftrightarrow x^7=y, \ x=y^7$

Dit geeft: $x^{49}=y^7=x$, dus $x^{49}-x=0, \ x(x^{48}-1)=0$,
$x=0, 1$ of -1.
$x=0 \Rightarrow y=0, \ x=1 \Rightarrow y=1, \ x=-1 \Rightarrow y=-1$.
Dit geeft drie punten $(0,0), (1,1), (-1,-1)$. Deze voldoen aan de vergelijkingen $\frac{\partial f}{\partial x}=0, \frac{\partial f}{\partial y}=0$, dus dit zijn de stationaire punten van f.

c) $\frac{\partial^2 f}{\partial x^2}=56x^6=A, \quad \frac{\partial^2 f}{\partial y \partial x}=-8=B, \quad \frac{\partial^2 f}{\partial y^2}=56y^6=C, \quad H=AC-B^2$

	A	B	C	H	classificatie
$(0,0)$	0	-8	0	$-56<0$	zadelpunt
$(1,1)$	56	-8	56	$56^2-(-8)^2>0$	minimum
$(-1,-1)$	56	-8	56	$56^2-(-8)^2>0$	minimum.

Er geldt: $f(1,1)=1-8+1=-6$, $f(x,y)\geq -6$ voor alle x,y
omdat kwadraten ≥ 0 zijn,
$f(-1,-1)=-6$,

Dus f neemt in $(1,1), (-1,-1)$ <u>absolute minima</u> aan.

3 a) $(3+2i)(5-i) = 3\cdot 5 - 3\cdot i + 2i\cdot 5 - 2i^2 = 15 - 3i + 10i + 2 = 17 + 7i$

$$\frac{(3+2i)(5-i)}{6+i} = \frac{17+7i}{6+i} = \frac{(17+7i)(6-i)}{6^2+1^2}$$

$$= \frac{17\cdot 6 - 17i + 7i\cdot 6 - 7i^2}{37} = \frac{102+25i+7}{37} = \boxed{\frac{109}{37}+\frac{25}{37}i}$$

b) Schrijf eerst $5\sqrt{3} - 5i$ in de vorm $r(\cos\varphi + i\sin\varphi)$.
Er geldt $r = |5\sqrt{3}-5i| = \sqrt{(5\sqrt{3})^2+(-5)^2} = \sqrt{75+25} = \sqrt{100} = 10$

$\cos\varphi = \frac{5\sqrt{3}}{10}$, $\sin\varphi = -\frac{5}{10} \Rightarrow \cos\varphi = \frac{1}{2}\sqrt{3}$, $\sin\varphi = -\frac{1}{2}$

We mogen $\varphi = -\frac{1}{6}\pi$ nemen, $5\sqrt{3}-5i = 10(\cos(-\frac{1}{6}\pi)+i\sin(-\frac{1}{6}\pi))$

Dus $(5\sqrt{3}-5i)^{171} = 10^{171}\left(\cos\left(-\frac{171}{6}\pi\right) + i\sin\left(-\frac{171}{6}\pi\right)\right)$

$= 10^{171}\left(\cos\left(-168\frac{1}{2}\pi\right) + i\sin\left(-168\frac{1}{2}\pi\right)\right)$

$= 10^{171}\left(\cos\left(-\frac{1}{2}\pi\right) + i\sin\left(-\frac{1}{2}\pi\right)\right) = \boxed{-10^{171}i}$

c) $iz^2 + (1+3i)z + 3 = 0 \Leftrightarrow z^2 + \frac{1+3i}{i}z + \frac{3}{i} = 0$ $\quad\left(\frac{1}{i} = -i\right)$

$\Leftrightarrow z^2 + (3-i)z - 3i = 0 \Leftrightarrow (z+3)(z-i) = 0$

$a = 3,\ b = -i$

$ab = -3i$

$\Leftrightarrow \boxed{z = -3 \text{ of } z = i}$

d) $z^{12} = 10^{12}i$ $\qquad 10^{12}i = 10^{12}\left(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi\right)$

$\Rightarrow z_k = \sqrt[12]{10^{12}}\left(\cos\left(\frac{1}{24}\pi + \frac{2k\pi}{12}\right) + \sin\left(\frac{1}{24}\pi + \frac{2k\pi}{12}\right)\right)$ $k=0,1,\dots,11$

$$\boxed{= 10\left(\cos\left(\frac{1}{24}\pi + \frac{k\pi}{6}\right) + i\sin\left(\frac{1}{24}\pi + \frac{k\pi}{6}\right)\right) \quad k=0,1,\dots,11}$$

④

④ a) $\sum_{n=0}^{\infty}\left(\frac{(-2)^n}{3^n}+2\cdot\frac{(-4)^n}{5^n}\right)=\sum_{n=0}^{\infty}\left(\frac{-2}{3}\right)^n+2\sum_{n=0}^{\infty}\left(\frac{-4}{5}\right)^n$

$=\frac{1}{1-(-\frac{2}{3})}+2\frac{1}{1-(-\frac{4}{5})}=\frac{1}{5/3}+\frac{2}{9/5}$

$=\frac{3}{5}+2\cdot\frac{5}{9}=\frac{3}{5}+\frac{10}{9}=\frac{27}{45}+\frac{50}{45}=\boxed{\frac{77}{45}}$

b) $\frac{3\sqrt[10]{n}+4}{n^2+1}=\frac{3\cdot n^{\frac{1}{10}}+4}{n^2+1}$ is van orde van grootte

$n^{\frac{1}{10}}/n^2=n^{\frac{1}{10}-2}=n^{-\frac{19}{10}}$

Dus we vergelijken $a_n:=\frac{3\sqrt[10]{n}+4}{n^2+1}$ met $b_n:=n^{-19/10}$

$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{3n^{\frac{1}{10}}+4}{n^2+1}\cdot n^{\frac{19}{10}}=\lim_{n\to\infty}\frac{3n^2+4\cdot n^{\frac{19}{10}}}{n^2+1}$

$=\lim_{n\to\infty}\frac{3+4n^{-1/10}}{1+n^{-2}}=3<\infty$

De reeks $\sum_{n=1}^{\infty}b_n=\sum_{n=1}^{\infty}n^{-19/10}$ convergeert want $\frac{19}{10}>1$

Volgens het vergelijkingskenmerk convergeert

$\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{3\sqrt[10]{n}+4}{n^2+1}$ ook.