The drift paradox in population dynamics

B.Sc. project in Applied Mathematics - Mathematical Biology (Applied Analysis)
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In various ecosystems a population of organism has to survive in an environment that introduces a strong directional bias in the spread of individuals. You may think of small sweet water shrimps that typically live on the bottom of a river (the 'benthos') but occasionally may be taken up by the flow. Or of bacteria in the intestines that are carried away by the food substances that are being digested. The 'drift paradox' in ecology is the idea that a population in such an environment should go extinct on the long run, because upstream territory could not be reoccupied. Nevertheless, stable population are observed in such environments.

The project consists of a literature study of the proposed hypothesis for resolution of this paradox, starting from the paper [1], in particular the various mathematical models that have been developed to support these solutions. Next, following the lines of reasoning in [1], the model developed there in terms of a system of two partial differential equations (PDEs) is analyzed in detail. One objective is to check the correctness of the proof of the main result (claimed to provide yet another solution to the drift paradox) and repair mistakes when found. If time permits, a model will be developed for a competing multi-species population of bacteria in the intestines and a start will be made with its analysis. This relates to recent experimental research on changes in the composition of the bacterial population in human intestines.

Knowledge of ordinary differential equations (Analysis 3) is required. Some acquaintance with PDEs, reaction-diffusion-advection equations in particular, is handy, but not required. You will learn the basic techniques for checking the (linear) stability of steady states during the project, if needed. You will learn to reason about the (complex) solutions of a cubic polynomial equation. Analytical techniques will be used. Performing numerical simulations is optional. Knowledge of (population) biology is not needed, but an interest in that topic is appreciated.

References:

[1] E. Pachepsky, F. Lutscher, R.M. Nisbet, M.A. Lewis (2005), Persistence, spread and the drift paradox, *Theoretical Population Biology* **67**, 61–73.

A detailed presentation of this project cannot be given in the last week of January, unfortunately. ***Please contact the lecturer by email if you are interested and want further information*.**