Spontaneous synchronization in complex networks

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Spontaneous synchronization is the process through which a population of coupled oscillators reaches a state of global synchronization without a centralized driving mechanism, but only through mutual interactions. Physical examples include "phase locking" in populations of coupled lasers, while biological examples include the synchronization of flashing fireflies and of cells in the heart and the brain.

Kuramoto [1] proposed a mathematical model of this process by considering a population of oscillators where the phase of each oscillator is coupled, via an interaction strength K, to the phases of the other oscillators in the system. Using a self-consistency argument, Kuramoto showed the existence of an order parameter distinguishing a "desynchronized" phase, where each oscillator proceeds at its own natural frequency, from a "synchronized" phase, where a finite fraction of all the oscillators becomes locked in phase. The phase transition occurs when the coupling strength exceeds a critical value K_c. In the mean-field version of the Kuramoto model, where each oscillator is subject to the simultaneous effect of all the other oscillators, exact mathematical results can be derived also for a number of extensions of the model, including the addition of stochastic noise, heterogeneous oscillators, etc.

However, the mathematical literature has not yet explored in detail the situation in which the interactions among the oscillators are arranged in general (and possibly complicated) networks. This setting has been studied mainly in the statistical physics literature [2], where the subject of complex networks has received significant attention over the last 15 years. When the network does not show any particular symmetry and has an intricate topology, the Kuramoto model (and its extensions) can lead to a rich and very interesting behavior, which is not simply captured by the synchronized/desynchronized dichotomy. In particular, if the network has a modular structure in which different so-called *communities* of nodes are more densely connected internally than with the rest of the network, then synchronization occurs over a hierarchy of different time scales (see Fig.1). This means that oscillators within each community synchronize first, then communities of communities synchronize, and so on. Each time scale depends on the size of the corresponding community, so the synchronization patterns depend on the details of the network structure and can lead to a rich phenomenology.

The aim of this project is to review the mathematical and physical literature about the Kuramoto model, extend the available mathematical results to more realistic network models and more types of noise, analyse empirical network data and run simulations of the model on all these structures. Thus, the project requires analytical and computational work, as well as data analysis.
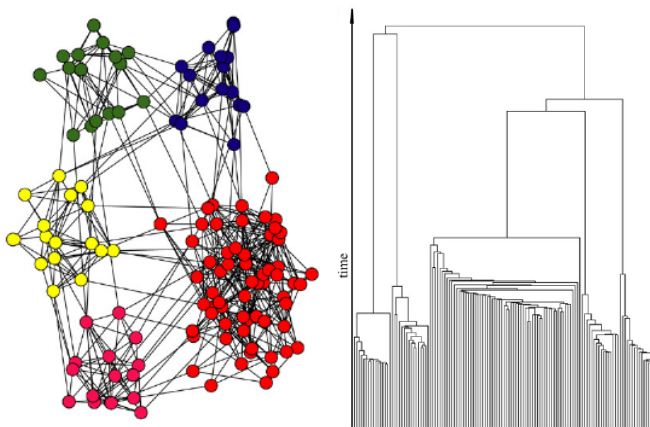

Figure 1. Hierarchical synchronization patterns on modular networks of coupled oscillators. Left: a network of oscillators with modular structure, where nodes of the same color form *communities* that are more densely connected internally than with the rest of network. Right: the resulting hierarchy of synchronization timescales (time is on the *y* axis; nodes are along the *x* axis and are indicated by the same color as in the left panel). Branching points indicate the time at which (groups of) nodes synchronize with each other.

[1] Kuramoto. *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984).
[2] Arenas et al. Synchronization in complex networks. *Physics Reports* **469**, 93-153 (2008).