Advanced Statistical Computing
Computational Statistics

Exercises for session 4

1. Triangular distribution. Simulate sums of pairs of independent uniformly distributed random variables. Study the histogram. Use your insight to develop an algorithm for drawing from a triangular distribution. The triangle has its peak at zero and its base runs from 0 to 1.

2. Sums of squares. Generate sums of squares of 5 independent normal variables. Compare their distribution to the appropriate \( \chi^2 \) distribution.

3. Kernel density estimation. Generate exponentially distributed random numbers. Use `density()` to plot the empirical density. Compare to the histogram. Do you notice a problem?

4. Rayleigh distribution. Simulate pairs of independent standard normal variables and interpret them as \( x \) and \( y \) of a vector. Compute the squared length of the vector and study its distribution.

5. Box-Müller. Take a look at example 4 at the beginning of Section 3.4. It describes the Box-Müller method for generating normally distributed numbers. Implement it in R.

6. Cauchy distribution. Draw 1000 sets of numbers from the Cauchy distribution. Do this for set size 2, 5, 10 and 20. Compute the median of each set (be smart, use a matrix and `apply()`). Study the distribution of the medians, for each set size.

7. Cauchy distribution again. Do the same as above, but do not compute medians of sets, but means. Do you notice anything special?

8. Acceptance-rejection sampling. In a formula for a density usually a numerical constant occurs, to make it a proper density, i.e. the integral is 1. In the standard normal density, \( \exp(-x^2/2) / \sqrt{2\pi} \), it is the factor \( 1/\sqrt{2\pi} \). Suppose I don’t know or forget this “normalizing constant”, how would that influence on acceptance-rejection sampling?