Advanced Statistical Computing

~~Computational Statistics~~

Exercises for week 4: Optimization

Exercise 1 The problem is to find the solution of the equation $x^2 = 2$ by an iterative method. Of course, the solution is $\sqrt{2}$, and the `R` command `print(sqrt(2),digits=20)` gives you this in many decimals.

a Implement the bisection algorithm for this problem. Choose starting values 0.1 and 10. How many iterations are needed to obtain accuracy 10^{-6}?

b Implement the method of Newton-Raphson for this problem. Choose starting value 10. How many iterations are needed to obtain accuracy 10^{-6}?

Exercise 2 Suppose that we observe a random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$ following the probit model:

$$\mathrm{P}_{\alpha,\beta}(Y = 1 | \, X = x) = \Phi(\alpha + \beta x) = 1 - \mathrm{P}_{\alpha,\beta}(Y = 0 | \, X = x),$$

where α, β are parameters and Φ is the standard normal distribution function. You can generate data and next fit this model with the `R` commands

```
> set.seed(1244)
> a=1; b=1; n=50
> x=rnorm(50)
> y=rbinom(50,size=1,prob=pnorm(a+b*x))
> data=data.frame(x=x,y=y)
> probreg=glm(y~x,family=binomial(link = "probit"),data=data)
> summary(probreg)
```

a Write your own program for fitting the maximum likelihood estimator using the function `optim` with the default method (Nelder-Mead).

b Compare your result to those obtained by `glm`. Take e.g. starting value $(0, 0)$

c Write your own program for fitting the maximum likelihood estimator, using the function `nlm`.

d Use the output for the Hessian to estimate the se's of the estimators. Compare this to the output obtained by `glm`.

Now suppose we want to change the probit link function to the Laplace one, i.e. we want to replace Φ by the cdf of the Laplace distribution

$$F(x) = \begin{cases} \frac{1}{2} e^x, & \text{if } x < 0, \\ 1 - \frac{1}{2} e^{-x}, & \text{if } x \geq 0. \end{cases}$$

This function is also contained as `plaplace` in `library(VGAM)`.

e Compute the MLE and an estimate for its covariance matrix. (Do not change the data, only the estimator!)