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# Week 2 homework.

24. The system is consistent, see Theorem 2  
(existence & uniqueness)

$$\underline{9} \quad x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{10} \quad x_1 \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -7 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}.$$

$$\underline{11} \quad \text{Row reduction of } A(\vec{v}) \text{ is } \left[ \begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Last column is not a pivot, so system is consistent,  
so  $\underline{b}$  is a linear combination of the  $\underline{\alpha_i}$ .

$$\underline{12} \quad \text{Row reduction of } A(\vec{v}) \text{ is } \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \frac{245}{33} \\ 0 & 1 & 0 & -4 \frac{1}{33} \\ 0 & 0 & 1 & -2 \frac{1}{11} \end{array} \right].$$

As for 11,  $\underline{b}$  is a linear combination of the  $\underline{\alpha_i}$ .

$$\underline{22} \quad \text{Any possibilities, e.g. } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

1 Undefined -  $A$  is  $3 \times 2$ , and  $\underline{b} \in \mathbb{R}^3$ .

2 Undefined -  $A$  is  $3 \times 1$ , &  $\underline{b} \in \mathbb{R}^2$ .

## 8 Matrix equation:

$$\begin{bmatrix} 2 & -1 & -4 & 0 \\ -4 & 5 & 3 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 5 \\ 12 \end{bmatrix} \quad (\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix})$$

14 Row reduce the matrix

$$\left[ \begin{array}{cccc|c} 2 & 5 & -1 & 1 & 4 \\ 0 & 1 & -1 & | & -1 \\ 1 & 2 & 0 & & 4 \end{array} \right]$$

to get

$$\left[ \begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & & 1 \end{array} \right]$$

System is inconsistent, so  $\underline{b}$  is not a linear combination of the columns of  $A$ , so is not in the span.

24 a) True; if  $A = [\underline{\alpha}_1, \dots, \underline{\alpha}_n]$  then the vector eqn is

$$x_1 \underline{\alpha}_1 + \dots + x_n \underline{\alpha}_n = \underline{b}$$

b) True; if  $A = [\underline{\alpha}_1, \dots, \underline{\alpha}_n]$ , can write

$\underline{b} = x_1 \underline{\alpha}_1 + \dots + x_n \underline{\alpha}_n$  some  $x_i \in \mathbb{R}$ ,  
so  $\underline{b}$  is a linear combination of the columns.

c) ~~True~~; Put the vectors as the columns of  $A$ ,  
& take  $\underline{x}$  as the weights (coefficients)  
of the linear combinations.

d) False, e.g.  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $\underline{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ , then  $A\underline{x} = \underline{b}$  is consistent.

e) True, See theorem 3.

f) False, eg take  $m = n = 1, A = [0]$ , and  $b = [1]$

30  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ . Every  $\leq$  in the span of the columns of  $A$  has a 0 in the final position.  
(Many possibilities).

$$\underline{3} \quad \begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}$$

$$\underline{4} \quad \begin{bmatrix} 3 \\ 8 \end{bmatrix}.$$

$$\begin{array}{l}
 \text{12} \left\{ \begin{array}{l}
 x_1 + x_4 = x_2 \\
 x_2 = x_3 + 100 \\
 x_3 + 80 = x_4 \\
 x_1 + 80 = 100
 \end{array} \right. \quad \left. \begin{array}{l}
 x_1 - x_2 + x_4 = 0 \\
 x_2 - x_3 = 100 \\
 x_3 - x_4 = -80 \\
 x_1 = 20
 \end{array} \right\} \rightarrow
 \end{array}$$

$$\left[ \begin{array}{cccc|c}
 1 & -1 & 0 & 1 & 0 \\
 0 & 1 & -1 & 0 & 100 \\
 0 & 0 & 1 & -1 & -80 \\
 1 & 0 & 0 & 0 & 20
 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c}
 1 & 0 & 0 & 0 & 20 \\
 0 & 1 & 0 & -1 & 20 \\
 0 & 0 & 1 & -1 & -80 \\
 0 & 0 & 0 & 0 & 0
 \end{array} \right]$$

$$\text{General solution } \underline{x} = \begin{bmatrix} 20 \\ 20 \\ -80 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R}$$

all  $x_i \geq 0 \Leftrightarrow t \geq 80$ . And  $x_4 = t$ ,

so smallest value for  $x_4$  is 80.