Week 2 solutions.

1) Is a basis; three pivots, so span has dimension 3, & is contained in $\mathbb{R}^3$, so equals $\mathbb{R}^3$.

2) Not a basis; $\text{Span}\{\underline{v}_1, \underline{v}_2, \underline{v}_3\} = \text{Span}\{\underline{v}_1, \underline{v}_3\}$, so has dimension ≤ 2.

3) Not a basis; row reduces to $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$, only 2 pivots, so $\dim(\text{Span}) \leq 2$.

4) Is a basis; row reduces to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, 3 pivots, so $\dim(\text{Span}) = 3$, so $= \mathbb{R}^3$.

23) $\{\underline{v}_1, \dots, \underline{v}_4\}$ contains a basis B, ~~but~~ but B has 4 elements since $\dim \mathbb{R}^4 = 4$, So actually $B = \{\underline{v}_1, \dots, \underline{v}_4\}$.

14) Say $a(1-t) + b(t-t^2) + c(2-t+t^2) = 1 + 3t - 6t^2$,

then
$$\begin{aligned} a + 2c &= 1 & &(\text{coeff of } t^0) \\ -a + b - c &= 3 & &(\text{coeff of } t^1) \\ -b + c &= -6 & &(\text{coeff of } t^2) \end{aligned}$$

Solve linear system:

$$\text{AEM} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 3 \\ 0 & -1 & 1 & -6 \end{array}\right] \xrightarrow{\text{RRed}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right] \qquad \begin{aligned} a &= 3 \\ b &= 5 \\ c &= -1 \end{aligned}$$

Coordinate vector is $\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

2 a) True, because the set contains a basis (with $\dim(V)$ elements)

b) True, because the set can be expanded to a basis (see 'Subspaces of a finite dimensional space')

c) True. Let B be a basis, with p elements, & let $\underline{v} \in V \setminus B$. Then $B \cup \{\underline{v}\}$ has $p+1$ elements & is a spanning set.

1) A row reduces to $\begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & 1 & -5/2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Col(A) has basis $\left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} \right\}$.

Nul(A): general solution is $\underline{x} = \begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} t$, $s, t \in \mathbb{R}$

A basis of Nul(A) is $\left\{ \begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$.