Read chapter 2.1, in particular
- the row-column rule for matrix multiplication;
- powers of a matrix
- the final part on the transpose of a matrix.
Answer the following questions, ready for the test on Friday.

In Exercises 1 and 2, compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let

\[
A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}
\]

1. \(-2A, B - 2A, AC, CD\)

2. \(A + 3B, 2C - 3E, DB, EC\)

In Exercises 5 and 6, compute the product \(AB\) in two ways: (a) by the definition, where \(Ab_1\) and \(Ab_2\) are computed separately, and (b) by the row-column rule for computing \(AB\).

5. \(A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}\)

7. If a matrix \(A\) is \(5 \times 3\) and the product \(AB\) is \(5 \times 7\), what is the size of \(B\)?

8. How many rows does \(B\) have if \(BC\) is a \(5 \times 4\) matrix?

9. Let \(A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}\) and \(B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}\). What value(s) of \(k\), if any, will make \(AB = BA\)?
11. Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \) and \( D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \). Compute \( AD \) and \( DA \). Explain how the columns or rows of \( A \) change when \( A \) is multiplied by \( D \) on the right or on the left. Find a \( 3 \times 3 \) matrix \( B \), not the identity matrix or the zero matrix, such that \( AB = BA \).

12. Let \( A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix} \). Construct a \( 2 \times 2 \) matrix \( B \) such that \( AB \) is the zero matrix. Use two different nonzero columns for \( B \).

Exercises 15 and 16 concern arbitrary matrices \( A \), \( B \), and \( C \) for which the indicated sums and products are defined. Mark each statement True or False. Justify each answer.

15. a. If \( A \) and \( B \) are \( 2 \times 2 \) matrices with columns \( \mathbf{a}_1 \), \( \mathbf{a}_2 \), and \( \mathbf{b}_1 \), \( \mathbf{b}_2 \), respectively, then \( AB = [ \mathbf{a}_1 \mathbf{b}_1 \quad \mathbf{a}_2 \mathbf{b}_2 ] \).

b. Each column of \( AB \) is a linear combination of the columns of \( B \) using weights from the corresponding column of \( A \).

c. \( AB + AC = A(B + C) \)

d. \( A^T + B^T = (A + B)^T \)

e. The transpose of a product of matrices equals the product of their transposes in the same order.
31. Show that $I_mA = A$ where $A$ is an $m \times n$ matrix. Assume $I_mx = x$ for all $x$ in $\mathbb{R}^m$.

32. Show that $AI_n = A$ when $A$ is an $m \times n$ matrix. [Hint: Use the (column) definition of $AI_n$.]
Define a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 5x_2 + 4x_3 \\ x_2 - 6x_3 \end{bmatrix}.$$ 

a) Write down the standard matrix for $T$.
b) Is $T$ injective?
c) Is $T$ surjective?