1. Is \( \lambda = 2 \) an eigenvalue of \[
\begin{bmatrix}
3 & 2 \\
3 & 8
\end{bmatrix}
\]? Why or why not?

2. Is \( \lambda = -3 \) an eigenvalue of \[
\begin{bmatrix}
-1 & 4 \\
6 & 9
\end{bmatrix}
\]? Why or why not?

3. Is \[
\begin{bmatrix}
1 \\
3
\end{bmatrix}
\] an eigenvector of \[
\begin{bmatrix}
1 & -1 \\
6 & -4
\end{bmatrix}
\]? If so, find the eigenvalue.

In Exercises 9–16, find a basis for the eigenspace corresponding to each listed eigenvalue.

9. \( A = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}, \lambda = 1, 3 \)
Find the eigenvalues of the matrices in Exercises 17 and 18.

17. \[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 3 & 4 \\
0 & 0 & -2
\end{bmatrix}
\]

18. \[
\begin{bmatrix}
5 & 0 & 0 \\
0 & 0 & 0 \\
-1 & 0 & 3
\end{bmatrix}
\]

In Exercises 21 and 22, \(A\) is an \(n \times n\) matrix. Mark each statement True or False. Justify each answer.

21. a. If \(A\mathbf{x} = \lambda\mathbf{x}\) for some vector \(\mathbf{x}\), then \(\lambda\) is an eigenvalue of \(A\).

b. A matrix \(A\) is not invertible if and only if 0 is an eigenvalue of \(A\).

c. A number \(c\) is an eigenvalue of \(A\) if and only if the equation \((A - cI)\mathbf{x} = 0\) has a nontrivial solution.

d. Finding an eigenvector of \(A\) may be difficult, but checking whether a given vector is in fact an eigenvector is easy.

e. To find the eigenvalues of \(A\), reduce \(A\) to echelon form.

24. Construct an example of a \(2 \times 2\) matrix with only one distinct eigenvalue.

25. Let \(\lambda\) be an eigenvalue of an invertible matrix \(A\). Show that \(\lambda^{-1}\) is an eigenvalue of \(A^{-1}\). [Hint: Suppose a nonzero \(\mathbf{x}\) satisfies \(A\mathbf{x} = \lambda\mathbf{x}\).]

26. Show that if \(A^2\) is the zero matrix, then the only eigenvalue of \(A\) is 0.

27. Show that \(\lambda\) is an eigenvalue of \(A\) if and only if \(\lambda\) is an eigenvalue of \(A^T\). [Hint: Find out how \(A - \lambda I\) and \(A^T - \lambda I\) are related.]