In Exercises 5–8, determine if the given set is a subspace of \( \mathbb{P}_n \) for an appropriate value of \( n \). Justify your answers.

5. All polynomials of the form \( p(t) = at^2 \), where \( a \) is in \( \mathbb{R} \).

6. All polynomials of the form \( p(t) = a + t^2 \), where \( a \) is in \( \mathbb{R} \).

7. All polynomials of degree at most 3, with integers as coefficients.

8. All polynomials in \( \mathbb{P}_n \) such that \( p(0) = 0 \).

9. Let \( H \) be the set of all vectors of the form \( \begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix} \). Find a vector \( \mathbf{v} \) in \( \mathbb{R}^3 \) such that \( H = \text{Span} \{ \mathbf{v} \} \). Why does this show that \( H \) is a subspace of \( \mathbb{R}^3 \)?

13. Let \( \mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \) and \( \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \).

a. Is \( \mathbf{w} \) in \( \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \)? How many vectors are in \( \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \)?

b. How many vectors are in \( \text{Span} \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \)?

c. Is \( \mathbf{w} \) in the subspace spanned by \( \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \)? Why?

In Exercises 3–6, find an explicit description of \( \text{Nul} \ A \), by listing vectors that span the null space.

3. \( A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix} \)

In Exercises 7–14, either use an appropriate theorem to show that the given set, \( W \), is a vector space, or find a specific example to the contrary.

7. \( \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 2 \right\} \)

8. \( \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 3r - 2 = 3s + t \right\} \)
Let $V$ and $W$ be vector spaces, and let $T : V \rightarrow W$ be a linear transformation. Given a subspace $U$ of $V$, let $T(U)$ denote the set of all images of the form $T(x)$, where $x$ is in $U$. Show that $T(U)$ is a subspace of $W$. 