Elliptic curves: exercise sheet 9

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Mastermath / DIAMANT, Fall 2013

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Exercise 21. Read page 42 of Silverman's The Arithmetic of Elliptic Curves (second edition) for the definition of the j-invariant of an elliptic curve. Let k be a field. Show that for every $x \in k$ there is an elliptic curve over k with j-invariant equal to x.

Exercise 22. Let k be a field of characteristic unequal to 2 containing i, a square root of -1. Let E be the elliptic curve given by $Y^2 = X^3 - X$ and let $[i] : E \longrightarrow E$ be the automorphism given by $(x, y) \mapsto (-x, iy)$. Compute formulas for the isogeny $\phi = \mathrm{id} + [i]$ and compute the coordinates of the points in $\ker \phi$.

Exercise 23. Exercise 3.30 of Silverman's book 'The Arithmetic of Elliptic Curves'.

Exercise 24. Exercise 3.32 of Silverman's book 'The Arithmetic of Elliptic Curves'.