Introduction to Algebraic Topology, Fall 2018, practice exercises - week 3

Exercise 1. Let Z be a connected and locally path connected topological space.

(i) Show that Z is path connected. Hint: let $w \in Z$ and let $E \subset Z$ be the subset of points $z \in Z$ such that there exists a path from w to z. Show that E is non-empty, open, and closed.

(ii) Let $p\colon Z \to X$ be a covering. Show that X is connected and locally path connected.

(iii) For those who like counterexamples: find a topological space that is path connected, but not locally path connected.

Exercise 2. (i) Let $p\colon Y \to X$ be a covering. Let $s\colon X \to Y$ be a continuous section of p, i.e. $s\colon X \to Y$ is a continuous map such that the equality $p \circ s = \mathrm{id}_X$ holds. Assume that Y is connected. Show that p is a homeomorphism with inverse s.

(ii) Let $p = \exp\colon \mathbb{C} \to \mathbb{C}^*$ be the covering from Exercise 1(iv) of Homework sheet I. Show that there does not exist a continuous section

$$s = \log\colon \mathbb{C}^* \to \mathbb{C}$$

of p.

(iii) Let $X = \mathbb{C}^* \backslash \mathbb{R}_{>0}$ and let $Y = p^{-1}X \subset \mathbb{C}$. Draw a picture of Y. Show that the restriction $r = p|_Y\colon Y \to X$ is a covering. Show that r is a trivial covering. Exhibit a continuous section $s\colon X \to Y$ of r. You have now constructed a "branch of the logarithm".

Exercise 3. Do Exercise 11.14 from Fulton's "Algebraic Topology: A first course". Compare with Exercise 127 from the Topologie syllabus of Fall 2017. Show that the 3-sheeted covering from Exercise 11.14 is not a G-covering.