The mathematics of change ringing

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This project involves some fun applications of elementary combinatorics and group theory.

Overview

Change ringing is a way of ringing bells that emerged in England in the 17th century. Bells hung for change ringing are widespread in England and also present in many other countries around the world. The object of change ringing is to ring a set of bells (typically 6 or 8) in a sequence of permutations ("rows"). Between each permutation and the next, any one bell is only permitted to move at most one place. The transition from one permutation to another is called a "change". For example, starting from the permutation 123456, it would be permitted to move to the permutation 214365, since each bell has only moved one place from its original position. Thus we can think of a graph whose vertices are permutations, and where two permutations are connected by an edge if it is possible to pass from one to another using a change. This is an example of a Cayley graph.

It is possible to put all 720 permutations of six bells into a sequence using only such changes: this is a Hamiltonian cycle on the Cayley graph, that is, a path which visits each vertex precisely once. However, a whole sequence of 720 changes might be difficult to remember, and so ringers prefer to construct a short, basic pattern (a "method") and then try to produce all permutations by repeating that pattern. This is a construction similar to covering the permutation group S_6 by cosets of a given subgroup, and it has been suggested that bell ringers were using some of the basic ideas of group theory over 100 years before what we think of as the first group theorists.

The student undertaking this project will write an account of the mathematics behind change ringing, relating it to combinatorics and group theory as learnt in the undergraduate syllabus.

References

[1] Wikipedia. Change ringing.

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[4] Richard G. Swan. A simple proof of Rankin's campanological theorem. *Amer. Math. Monthly* **106** (1999), no. 2, 159–161.