Distribution of the number of points on biquadratic curves

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We will start with the definition of a biquadratic curve.

Definition 0.1. *A biquadratic curve is a curve in the projective space $\mathbb{P}^3$ defined by a set of equations like this*

$$C : \begin{cases} y_1^2 = f(x)h(x) \\ y_2^2 = g(x)h(x) \\ y_3^2 = f(x)g(x) \end{cases},$$

where $f(x), g(x)$ and $h(x)$ are pairwise coprime polynomials. We denote by B_{n_1,n_2,n_3} the set of biquadratic curves defined by polynomials $f(x), g(x)$ and $h(x)$ of degree equal to n_1, n_2 and n_3.

Looking at the points of these curves over a finite field $\mathbb{F}_q$ (where $q = p^r$) is equivalent to looking at the simultaneous solutions of the three equations. Let us denote the extended quadratic character modulo $\mathbb{F}_q$ by $\chi : \mathbb{F}_q \to \{\pm 1, 0\}$ (that is, $\chi(r) = 0$ iff p divides r and $\chi(r) = \pm 1$, if p does not divide r acoording to r being a square in $\mathbb{F}_q$ or not). Then,

$$\#C(\mathbb{F}_q) = \sum_{r \in \mathbb{F}_q} (\chi(f(r)h(r)) + \chi(g(r)h(r)) + \chi(f(r)g(r)) + 1).$$

The formula above comes from the fact that $(\chi(f(r)h(r)), \chi(f(r)h(r)), \chi(f(r)g(r)))$ can (up to permutation) only take the values $(1,1,1)$, $(1,0,0)$, $(1,-1,-1)$ or $(-1,0,0)$ that correspond to the cases in which there are 4, 2, 0 or 0 solutions with $x = r$.

In this project, we will study the distribution of $\#C(\mathbb{F}_q)$ when we vary C inside the family B_{n_1,n_2,n_3}.

References

E. Lorenzo, G. Meleleo, P. Milione, Statistics for biquadratic covers of the projective line over finite fields, http://arxiv.org/abs/1503.03276.