The Lefschetz fixed point theorem.

The Lefschetz fixed point theorem in toplogy is a far-reaching generalisation of Brouwer's fixed point theorem.

Let X be a topological space that is a retract of a finite simplicial complex (this includes all compact manifolds). One can show that the singular homology groups with rational coefficients $H_k(X, \mathbb{Q})$ are finite dimensional $\mathbb{Q}$ vector spaces.

If $f : X \longrightarrow X$ is a continuous map, the maps $f_* : H_k(X, \mathbb{Q}) \longrightarrow H_k(X, \mathbb{Q})$ induced by f are linear endomorphisms. Set

$$\Lambda_f := \sum_{k \geqslant 0} (-1)^k \mathrm{Tr}(f_* : H_k(X, \mathbb{Q}) \longrightarrow H_k(X, \mathbb{Q})),$$

the alternating sum of the traces of these endomorphisms. The celebrated Lefschetz fixed point theorem states that if Λ_f is nonzero then f has at least one fixed point.

We propose to study the proof of this theorem. As it makes use of simplicial complexes and simplicial approximation, it would also be the occasion to learn about CW complexes which are ubiquitous and provide a lot of interesting examples in algebraic topology.

Let us mention that Lefschetz-type theorems also appear in many areas of modern mathematics. Most notably, Lefschetz principles in étale cohomology played a major role in proving the Weil conjectures.