Resistances in families of electrical networks

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Abstract

The aim of the project is to understand good notions of a 'family of graphs', and to look into how electrical resistances vary in such families.

1 Basic definitions

I do not claim that the following foundations are optimal (or even good), if you can do better then great!

A *graph* is a pair (G, V) where:

1. G is a compact connected metric space such that for every point $g \in G$ there exists an open neighbourhood U_g of g, an integer $n > 0$, a real number $\epsilon > 0$ and an isometry
$$U_g \to \{re^{2\pi id/n} | r \in [0, \epsilon), 0 \leq d \leq n\} \subset \mathbb{C}$$
where the latter has the Euclidean metric. We call the integer n the 'valence' of g. It is not hard to show that it is well-defined.

2. V is a set of points of G which include all points of valence not equal to 2 (we refer to the latter as 'fundamental vertices').

Question: is the set of fundamental vertices always finite? Probably... In any case, it is probably good to require that the set of vertices is finite.

An *edge* of G is a connected component of $G \setminus V$. The length of an edge e of G is defined by
$$\text{len}(e) = \sup\{d(a, b) | a, b \in e\}.$$
Note that the length of an edge need not equal the distance between its endpoints.

It seems easy to define an automorphism of graphs, though to define a morphism seems a little more delicate.

2 Families of graphs

Possible definition: let M be a metric space, $G \to M$ a map of metric spaces, and V a set of sections. We say $(G/M, V)$ is a *family of graphs* if for every $m \in M$, the fibre (G_m, V_m) is a graph in the above sense.

Question: can you make a good moduli space of metrised graphs?

Possible references:

1. 'Tropical curves and metric graphs', by, Melody Tung Chan, see `http://www.math.harvard.edu/~mtchan/thesis_mchan.pdf`.

2. 'Introduction to algebraic stacks' by Kai Behrend, in 'Moduli Spaces', edited by Leticia Brambila-Paz, Peter Newstead, Richard P. Thomas, Oscar Garca-Prada. Book DOI: `http://dx.doi.org/10.1017/CBO9781107279544`.

3 Resistances

If G is a graph then we can think of G as an electrical network where an edge e of length l is thought of as a resistor of resistance l. If p, q are two vertices of G then one can ask for the 'effective resistance' between, p and q. There is a beautiful combinatorial formula for this in terms of the edges, see `http://arxiv.org/abs/1412.8207`, page proposition 9.1 (this will probably be explained in my talk?).

If now $(G/M, V)$ is a family of graphs and p, q are two sections (maybe in V), then for each m we can compute the resistance between p_m and q_m. This gives a function from M to $\mathbb{R}$.

Question: When is this function *continuous*? I think it should be possible to answer this question using results from `http://arxiv.org/abs/1412.8207`... What happens in the universal case?