Finite rings with abelian unit groups

This is a project for a student who likes algebra, especially the theory of not necessarily commutative rings.

Clearly, the unit group of any commutative ring is abelian. but there are many non-commutative rings that also have an abelian unit group. As an example, one may consider the ring of upper-triangular 2×2-matrices over the field $\mathbf{F}_2$ of 2 elements.

The main purpose of the project is to classify all *finite* rings of which the unit group is abelian. (Note that the example just given is finite.) There is a sense in which finite rings have "many" units, and this is what should make such a classification possible.

In the course of the project, the student will need to learn several basic results and techniques from non-commutative ring theory. These include the theory of the Jacobson radical, the structure of semisimple rings, the subject of artinian rings, and properties of idempotents in rings.

It is very likely that the techniques to be used will apply to a larger class of rings than just the finite ones, for example the class of "left artinian" rings. Maybe the student can decide whether the results also apply to the much larger class of "semilocal" rings.

Supervisor: Hendrik Lenstra.