Ex. 10.1. Suppose that $A$ is a symmetric $2 \times 2$ matrix of determinant 2 for which $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is an eigenvector with eigenvalue $-1$.

1. What is the other eigenvalue of $A$?
2. What is the other eigenspace?
3. Determine $A$.

Ex. 10.2. Consider the quadratic form $q(x, y) = 11x^2 - 16xy - y^2$.

1. Find a symmetric matrix $A$ for which $q(x, y) = (x \ y) \cdot A \cdot (x \ y)$.
2. Find real numbers $a, b$ and an orthogonal map $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ so that $q(f(u, v)) = au^2 + bv^2$ for all $u, v \in \mathbb{R}$.
3. What values does $q(x, y)$ assume on the unit circle $x^2 + y^2 = 1$?

Ex. 10.3. What values does the quadratic form $q(x, y, z) = 2xy + 2xz + y^2 - 2yz + z^2$ assume when $(x, y, z)$ ranges over the unit sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$?

Ex. 10.4. Suppose that $A$ is an anti-symmetric $n \times n$ matrix over the real numbers.

1. Show that every eigenvalue of $A$ over the complex numbers lies in $i\mathbb{R}$.
2. If $n$ is odd, show that 0 is an eigenvalue of $A$. 