Linear algebra 2: exercises for Section 3

Ex. 3.1. Let A be a nilpotent $n \times n$ matrix. Show that $\mathrm{id}_n + A$ is invertible.

Ex. 3.2. Let A be a nilpotent $n \times n$ matrix. Show that $A^n = 0$.

Ex. 3.3. Let N be a 9×9 matrix for which $N^3 = 0$. Suppose that N^2 has rank 3. Prove that N has rank 6.

Ex. 3.4. Let N be a 12×12 matrix for which $N^4 = 0$.

1. Show that the kernel of N^2 contains the image of N^2.
2. Show that the rank of N is at most 9.
3. Show that the rank of N is equal to 9 if the kernel of N^2 is equal to the image of N^2.

Ex. 3.5. For which $x \in R$ is the following matrix nilpotent?

$$\begin{pmatrix} 2x & x & -1 \\ -4 & -1 & -3 \\ 5 & 2 & 3 \end{pmatrix}$$

Ex. 3.6. For each of the matrices

$$\begin{pmatrix} 4 & -4 & 12 \\ 1 & -1 & 3 \\ -1 & 1 & -3 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 8 \\ 0 & 1 & 1 \\ -1 & 1 & -3 \end{pmatrix}$$

give a basis of $\mathbb{R}^3$ for which the matrix sends each basis vector either to 0 or to the next basis vector in the basis.

Ex. 3.7. Do the same for the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -5 & -2 & 2 & -1 \\ -3 & 0 & 2 & -1 \\ -5 & -2 & 2 & -1 \end{pmatrix}$$