**Linear algebra 2: Homework set 4**

**Due date: November 29 13:45**

**(HW 4.1).** Let $V = \mathbb{C}^2$ be the standard complex vector space of dimension 2. Consider the map $\phi\colon V \times V \to \mathbb{C}$ given by $\phi((z_1, z_2), (w_1, w_2)) = z_1 w_2 + z_2 w_1$

1. Is $\phi$ bilinear? Is $\phi$ sesquilinear? Is $\phi$ symmetric? Is $\phi$ hermetian? Is $\phi$ an inner product? Motivate your answers.
2. Give a basis $v_1, v_2$ of $V$ so that $\phi(v_i, v_j) = 1$ if $i = j$ and $\phi(v_i, v_j) = 0$ if $i \neq j$

**(HW 4.2).** Let $V$ be the 2-dimensional subspace of $\mathbb{R}^3$ given by $x_1 + x_2 + 2x_3 = 0$. The standard inner product on $\mathbb{R}^3$ restricts to an inner product on $V$. Give an orthonormal basis of $V$ for this inner product.

**(HW 4.3).** Let $V = \mathbb{R}^3$ be the standard 3-dimensional vector space over $\mathbb{R}$, and let $\phi$ be the symmetric bilinear map $\phi\colon V \times V \to \mathbb{R}$ which on the standard basis is given by the matrix

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

1. Compute the determinant of the matrix.
2. Is $\phi$ positive definite?
3. What is the signature of $\phi$?
4. Same three questions when $\phi$ is given by

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$