Linear algebra 2: Homework set 5

Due date: December 13 13:45

(HW 5.1). Consider $\mathbb{C}^3$ with the statandard hermetian inner product $\langle \cdot, \cdot \rangle$, and let $v \in \mathbb{C}^3$ be a vector with $\langle v, v \rangle = 1$. Define the linear map $f\colon \mathbb{C}^3 \to \mathbb{C}^3$ by $f(x) = x - i\langle x, v \rangle v$.

1. Show that the adjoint of f is given by $f^*(x) = x + i\langle x, v \rangle v$.
2. Find the eigenvalues and eigenvectors of f.
3. Show that f is normal.
4. If f an isometry?

(HW 5.2). Give the Jordan normal form of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 5 & 2 & -2 \\ 0 & -4 & 0 & 4 \end{pmatrix}$$

(HW 5.3). Let the quadratic form $q\colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$q(\begin{pmatrix} x \\ y \end{pmatrix}) = 3x^2 - 4xy.$$

1. Give a symmetric matrix A for which
$$q(\begin{pmatrix} x \\ y \end{pmatrix}) = (x\ y) \cdot A \cdot \begin{pmatrix} x \\ y \end{pmatrix}.$$
2. Find $a, b \in \mathbb{R}$ and an orthogonal 2×2-matrix C so that $q(C \begin{pmatrix} u \\ v \end{pmatrix}) = au^2 + bv^2$ for all $u, v \in \mathbb{R}$.
3. What values does q assume on the unit circle in $\mathbb{R}^2$?