

# WEB CALCULI IN REPRESENTATION THEORY

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ABSTRACT. A striking question is if one can present the category  $\mathbf{Mod}(\mathfrak{g})$  of finite-dimensional representations of some Lie algebra  $\mathfrak{g}$ , via *generators and relations* – or even better: via *diagrammatic* generators and relations.

The question itself is very hard and only partial solutions are known: it goes back to work of Schur and Brauer that the subcategory of  $\mathbf{Mod}(\mathfrak{g})$  tensor generated by the vector representation can be (almost) described if  $\mathfrak{g}$  is of classical type and certain dimensions are “big enough”. Moreover, Rumer, Teller and Weyl showed (more or less) already in the 30ties that the Temperley-Lieb algebra can be seen as a diagrammatic realization of the representation category of  $\mathfrak{sl}_2$ -modules tensor generated by the vector representation of  $\mathfrak{sl}_2$  – providing a topological (and fun!) tool to study the latter.

In this talk I explain “howe” one can prove such a (*diagrammatic*) realization.

Everything in this talk can be quantized (roughly: put quantum brackets everywhere) and everything is amenable to categorification, but *I will keep it easy* and explain the  $\mathfrak{sl}_2$ -case in details and give some applications to knot theory of the whole calculus.