Abstract: The path $W[0,t]$ of a Brownian motion on a d-dimensional torus $\mathbb{T}^d$ run for time t is a random compact subset of $\mathbb{T}^d$. In this talk we look at the geometric properties of the complement $C(t) = \mathbb{T}^d \setminus W[0,t]$ as $t \to \infty$ for $d \geq 3$. Questions we address are the following:

1. What is the linear size of the largest region in $C(t)$?
2. What does $C(t)$ look like around this region?
3. Does $C(t)$ have some sort of 'component-structure'?
4. What are the largest capacity, largest volume and smallest principal Dirichlet eigenvalue of the components of $C(t)$?

We speculate about what happens for $d = 2$, which is much harder to understand.