STELLINGEN

behorende bij het proefschrift
On p-adic Decomposable Form Inequalities
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Let $F = L_1 \dots L_d \in \mathbb{Z}[X_1, \dots, X_n]$ be a decomposable form of degree d where $L_1, \dots, L_d \in \overline{\mathbb{Q}}[X_1, \dots, X_n]$ are linear forms. Define

$$a(F) := \max_{\substack{\mathcal{L} \subsetneq \{\mathbf{L}_1, \dots, \mathbf{L}_d\} \\ 0 < \text{rank } (\mathcal{L}) < n}} \frac{|\mathcal{L}|}{\text{rank } (\mathcal{L})}.$$

Let $S = \{\infty, p_1, \dots, p_r\}$ be a finite subset of $M_{\mathbb{Q}}$. Define

$$\mathbb{A}_{F,S}(m) := \Big\{ (\mathbf{x}_p)_p \in \prod_{p \in S} \mathbb{Q}_p^n : \prod_{p \in S} |F(\mathbf{x}_p)|_p \le m \ , \ |\mathbf{x}_p|_p = 1 \text{ for } p \in S_0 \Big\},$$

$$N_{F,S}(m) := \Big| \Big\{ \mathbf{x} \in \mathbb{Z}^n : \prod_{p \in S} |F(\mathbf{x})|_p \le m \ , \ \gcd(x_1, x_2, \dots, x_n, p_1 \cdots p_r) = 1 \Big\} \Big|.$$

Let μ_∞ be the normalized Lebesgue measure on $\mathbb{R}$ with $\mu_\infty([0,1]) = 1$ and μ_p be the normalized Haar measure on $\mathbb{Q}_p$ with $\mu_p(\mathbb{Z}_p) = 1$. On $\prod_{p \in S} \mathbb{Q}_p^n$, we define the product measure $\mu^n = \prod_{p \in S} \mu_p^n$. The $C_1(\cdot), \dots, C_6(\cdot)$ below are constants which depend only on the indicated parameters.

1. Suppose $F(\mathbf{x}) \neq 0$ for every non-zero $\mathbf{x} \in \mathbb{Z}^n$ and $a(F) < \frac{d}{n}$. Then

   $$\mu^n(\mathbb{A}_{F,S}) \le C_1(n, d, S).$$

   (Theorem 2.13)

2. Assume the same conditions above. Let $\mathbb{A}_{F,p} = \{\mathbf{x} \in \mathbb{Q}_p^n : |F(\mathbf{x})|_p \le 1\}$. Then

   $$\mu_p^n \left(\mathbb{A}_{F,p}\right) \le C_2(n, d, p).$$

3. Suppose $F(\mathbf{x}) \neq 0$ for every non-zero $\mathbf{x} \in \mathbb{Z}^n$. Also suppose $a(F|_T) < \frac{d}{\dim T}$ for every linear subspace T of dimension at least 2 of $\mathbb{Q}^n$. Then

   $$|N_{F,S}(m) - \mu^n(\mathbb{A}_{F,S}(m))| \le C_3(n, d, S, F) m^{\frac{n}{d+n-2}} \text{ as } m \to \infty.$$

   (Theorem 2.14)

4. Assume the same conditions above. If $\gcd(n, d) = 1$, then

   $$|N_{F,S}(m) - \mu^n(\mathbb{A}_{F,S}(m))| \le C_4(n, d, S) m^{\frac{n}{d+n-2}} \text{ as } m \to \infty.$$

   (Theorem 4.1.1 and Theorem 5.0.3)

5. Let $F_1, \ldots, F_r \in \mathbb{R}[X_1, \ldots, X_n]$. Suppose that F_i has total degree d_i for $i = 1, \ldots, r$. Let $B, m_1, \ldots, m_r$ be positive reals and

$$\mathcal{A} := \{\mathbf{x} \in \mathbb{R}^n : |F_1(\mathbf{x})| \leq m_1, \ldots, |F_r(\mathbf{x})| \leq m_r\}.$$

   Assume $\mathcal{A} \subseteq \{\mathbf{x} \in \mathbb{R}^n : |\mathbf{x}|_{sup} \leq B\}$. Then

$$\left|\mu_\infty^n(\mathcal{A}) - |\mathcal{A} \cap \mathbb{Z}^n|\,\right| \leq n2^r d_1 \cdots d_r \cdot (2B+1)^{n-1}.$$

   (Lemma 1.4.1)

   We say that two decomposable forms $F, G \in \mathbb{Z}[X_1, \ldots, X_n]$ are *S-equivalent* if there exist $t \in \mathbb{Z}_S^*$ and $T \in \mathrm{GL}_n(\mathbb{Z}_S)$ such that $G = t \cdot F_T$.

6. Let F be a decomposable form of degree $n+1$. There exists a decomposable form G in the S-equivalent class of F such that

$$\mathcal{H}(G) \leq C_5(n, S) \left(\prod_{p \in S} |D(G)|_p\right)^{\frac{2}{n+1}}.$$

   (See $(1.1.1)$ for the definition of $\mathcal{H}(G)$ and Definition $4.2.2$ for $D(G)$.)

   Let p be a prime number. We say that F, G are $\mathrm{GL}_n(\mathbb{Q}_p)$-equivalent if there exists $t \in \mathbb{Q}_p^*$ and $T \in \mathrm{GL}_n(\mathbb{Q}_p)$ such that $G = t \cdot F_T$.

7. The collection of decomposable forms $F \in \mathbb{Q}_p[X_1, \ldots, X_n]$ of degee $n+1$ with $D(F) \neq 0$ is a union of finitely many $GL_n(\mathbb{Q}_p)$-equivalence classes.

8. Let F be a decomposable form of degree $n+1$ with $D(F) \neq 0$. Then

$$\left(\prod_{p \in S} |D(F)|_p\right)^{\frac{1}{2(n+1)}} \cdot \mu^n(\mathbb{A}_{F,S}) \leq C_6(n, S)$$

9. Voor iets hoort iets. No pains no gains.