Finite Dimensional Pure Motives

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Abstract

In his paper [4] S.-I. Kimura introduced a notion of finite dimensionality for Chow groups and he proved that the Chow group of a product of smooth projective curves is finite dimensional.

In the 90's, P.O'Sullivan and him conjectured independently that this should hold for any smooth projective variety X. Namely, there should exist a canonical decomposition

$$\mathsf{CH}^*(X)_{\mathbb{Q}} = \mathsf{CH}^*(X)_+ \oplus \mathsf{CH}^*(X)_-$$

in an even and an odd part.

A suitable framework to develop such an idea is the theory of motives where the question posed by Kimura and O'Sullivan naturally links to other conjectures in the field (e.g.: Voevodsky or Bloch-Beilinson conjectures).

In this talk we will give a brief exposition of Grothendieck's construction of pure motives, following [2]. Then we will introduce the related notion of Schur finite-dimensionality (see for example [5],[3]) as well as Kimura-O'Sullivan finite dimensionality in abstract pseudo-abelian tensor categories and apply it to our framework ([1]). Finally, we will present some examples and applications.

References

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