APPROXIMATION OF COMPLEX ALGEBRAIC NUMBERS BY ALGEBRAIC NUMBERS OF BOUNDED DEGREE

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Define the height $H(P)$ of $P \in \mathbb{Z}[X]$ to be the maximum of the absolute values of the coefficients of P. Further, for an algebraic number ξ, define its height $H(\xi)$ to be the height of the minimal polynomial of ξ. For $\xi \in \mathbb{C}$ and for a positive integer n, denote by $w_n(\xi)$ the supremum of all reals w with the property that there are infinitely many polynomials $P \in \mathbb{Z}[X]$ of degree at most n such that $0 < |P(\xi)| \leq H(P)^{-w}$. Further, define $w_n^*(\xi)$ to be the supremum of all reals w^* such that there are infinitely algebraic numbers $\alpha \in \mathbb{C}$ such that $|\xi - \alpha| \leq H(\alpha)^{-w^*-1}$. The functions w_n and w_n^* were introduced by Mahler and Koksma, respectively, and they play an important role in the classification of transcendental numbers.

It is known that $w_n(\xi) = w_n^*(\xi) = \min(n-1, d)$ for every real algebraic number ξ of degree d. This result is a consequence of W.M. Schmidt's celebrated Subspace Theorem from Diophantine approximation. As it turns out, the problem to compute $w_n(\xi)$, $w_n^*(\xi)$ for complex, non-real algebraic numbers ξ is more complicated. In my talk, I discuss some joint work with Yann Bugeaud in this direction, in which we determined $w_n(\xi)$, $w_n^*(\xi)$ for all complex, non-real algebraic numbers ξ of degree $\leq n+2$ or $\geq 2n-1$.