Reduction of binary forms of given discriminant and root separation of irreducible polynomials

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We consider binary forms $F \in \mathbb{Z}[X,Y]$ of given degree $n \geq 2$ and given discriminant $D(F) = D$. Two binary forms $F, G \in \mathbb{Z}[X,Y]$ are called $\mathrm{GL}(2,\mathbb{Z})$-equivalent if $G(X,Y) = F(aX+bY, cX+dY)$ for some matrix $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{GL}(2,\mathbb{Z})$. It is well-known, that two $\mathrm{GL}(2,\mathbb{Z})$-equivalent binary forms have the same discriminant. A binary form having minimal height in its $\mathrm{GL}(2,\mathbb{Z})$-equivalence class is called *reduced*. We are interested in the problem of estimating the height of a reduced binary form in terms of its discriminant. It is conjectured, that if $F \in \mathbb{Z}[X,Y]$ is a reduced binary form of degree n and discriminant $D(F) = D \neq 0$, then for the height $H(F)$ of F we have (*) $H(F) \leq c_1(n)|D|^{c_2(n)}$. For $n = 2, 3$, classical theories of Gauss and Hermite give bounds of this type, but for $n \geq 4$ only weaker results have been proved. I will give an overview of the existing results. Further, I will discuss applications to root separation of irreducible polynomials.

Lastly, I will discuss recent work of my PhD-student Weidong Zhuang, who proved a function field analogue of conjecture (*) for reduced binary forms.