UITWERKING 1E HERKANSING CONTINUE WISKUNDE 2
30/6/2021

1 a) De inhoud van het omwentelingslichaam is

$$\int_0^{\pi/6} \pi f(x)^2\,dx = \int_0^{\pi/6} \pi(\sin x + \cos x)\,dx = \left[\pi(-\cos x + \sin x)\right]_0^{\pi/6}$$

$$= \pi\left(-\tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}\right) - \pi(-1+0) = \boxed{\pi\left(\tfrac{3}{2} - \tfrac{1}{2}\sqrt{3}\right)}$$

b) Partiële integratie:

$$\int (2x+3\sqrt{x})\ln x\,dx = \int (\ln x)\cdot(2x+3\sqrt{x})\,dx =$$

$$= (\ln x)\cdot(x^2+2x^{3/2}) - \int (x^2+2\sqrt{x^3})\cdot\frac{1}{x}\,dx$$

$\ln' x = \frac{1}{x}$

$f(x) = \ln x$
$g'(x) = 2x+3\sqrt{x}$
$g(x) = x^2+2x^{3/2}$

$$= (\ln x)(x^2+2x^{3/2}) - \int (x+2x^{1/2})\,dx = \boxed{(\ln x)\cdot(x^2+2x^{3/2}) - \left(\tfrac{1}{2}x^2+\tfrac{4}{3}x^{3/2}\right)+C}$$

c)

$$\int \frac{x\,dx}{(x^2+3)^{5/2}} = \int \frac{\frac{1}{2}du}{u^{5/2}} = \frac{1}{2}\left(-\frac{2}{3}u^{-3/2}\right)+C = -\frac{1}{3}u^{-3/2}+C$$

$$= -\frac{1}{3}(x^2+3)^{-3/2}+C$$

$u = x^2+3$
$du = 2x\,dx$
$x\,dx = \frac{1}{2}du$

Dus

$$\int_0^{\infty} \frac{x\,dx}{(x^2+3)^{5/2}} = \lim_{B\to\infty} \int_0^B \frac{x\,dx}{(x^2+3)^{5/2}} = \lim_{B\to\infty} \left[-\frac{1}{3}(x^2+3)^{-3/2}\right]_0^B$$

$$= \lim_{B\to\infty} \underbrace{-\frac{1}{3}(B^2+3)^{-3/2}}_{\to\, 0} + \frac{1}{3}\cdot 3^{-3/2} = \boxed{3^{-5/2}}$$

2 a) $f(x,y) = 4x^3 - 3y^4 + 6x^2y^2$

$\frac{\partial f}{\partial x} = 12x^2 + 12xy^2 = 12x(x+y^2)$, $\frac{\partial f}{\partial y} = -12y^3 + 12x^2y = 12y(-y^2+x^2)$

$\frac{\partial f}{\partial x} = 0$ impliceert $x=0$ of $x=-y^2$

$x=0$ invullen in $\frac{\partial f}{\partial y} = 0$ geeft $-12y^3 = 0$ dus $y=0$

Dit geeft het stationaire punt $(0,0)$.

$x=-y^2$ invullen in $\frac{\partial f}{\partial y} = 0$ geeft $12y(-y^2+y^4) = 0$

dus $12y^3(-1+y^2) = 0$, dus $y=0$ of $y^2 = 1$ dus $y=0, 1$ of -1.

Als we dit combineren met $x=-y^2$ krijgen we de punten $(0,0)$, $(-1,1)$, $(-1,-1)$. Dit zijn de drie stationaire ~~Als je~~ punten van f

b) $\frac{\partial^2 f}{\partial x^2} = 24x + 12y^2 = A$, $\frac{\partial^2 f}{\partial y \partial x} = 24xy = B$, $\frac{\partial^2 f}{\partial y^2} = -36y^2 + 12x^2 = C$

$H = AC - B^2$

Classificaties: $A>0, H>0$ minimum; $A<0, H>0$ maximum; $H<0$: zadelpunt; $H=0$: geen uitsluitsel

	A	B	C	H	
(0,0)	0	0	0	0	geen uitsluitsel
(-1,1)	-12	-24	-24	$12\times24 - 24^2 = -288 < 0$	zadelpunt
(-1,-1)	-12	24	-24	$-288 < 0$	zadelpunt

c) $f(x,0) = 4x^3$. Er geldt: $f(0,0) = 0$, $f(x,0) > 0$ voor $x>0$, $f(x,0) < 0$ voor $x<0$. Dus f neemt in $(0,0)$ geen maximum of minimum aan, $(0,0)$ is een zadelpunt van f

d) vgl raakvlak: $z = f(1,1) + \frac{\partial f}{\partial x}(1,1)(x-1) + \frac{\partial f}{\partial y}(1,1)(y-1)$

$f(1,1) = 4-3+6 = 7$, $\frac{\partial f}{\partial x}(1,1) = 12+12 = 24$, $\frac{\partial f}{\partial y}(1,1) = -12+12 = 0$

De vergelijking wordt: $\boxed{z = 7 + 24(x-1) = 24x - 17}$

3 a) $i^2=-1$, $i^4=(-1)^2=1$, $i^5=i^4.i=i$

$$\frac{i^5(3+i)}{5+4i}=\frac{i(3+i)}{5+4i}=\frac{(3i+i^2)(5-4i)}{(5+4i)(5-4i)}=\frac{(-1+3i)(5-4i)}{5^2+4^2}$$

$$=\frac{-1.5+(-1)(-4i)+(3i).5-3.4.i^2}{41}=\frac{-5+4i+15i+12}{41}$$

$$=\frac{7+19i}{41}=\boxed{\frac{7}{41}+\frac{19}{41}i}$$

b) Schrijf eerst $8-8i$ in de vorm $r(\cos\varphi+i\sin\varphi)$

$r=|8-8i|=\sqrt{8^2+(-8)^2}=8\sqrt{1+1}=8\sqrt{2}$

$\cos\varphi=\frac{8}{8\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{1}{2}\sqrt{2}$, $\sin\varphi=\frac{-8}{8\sqrt{2}}=\frac{-1}{\sqrt{2}}=-\frac{1}{2}\sqrt{2}$

We kunnen $\varphi=-\frac{1}{4}\pi$ nemen $8-8i=8\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi\right)+i\sin\left(-\frac{1}{4}\pi\right)\right)$

$$(8-8i)^{81}=(8\sqrt{2})^{81}\left(\cos\left(-\frac{81}{4}\pi\right)+i\sin\left(-\frac{81}{4}\pi\right)\right)$$

$$=(8\sqrt{2})^{81}\left(\cos\left(-\frac{1}{4}\pi\right)+i\sin\left(-\frac{1}{4}\pi\right)\right)=(8\sqrt{2})^{81}\left(\frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt{2}\,i\right)$$

$$=(8\sqrt{2})^{81}\cdot\frac{1}{2}\sqrt{2}(1-i)=2^{\frac{7}{2}\cdot 81}\cdot 2^{-\frac{1}{2}}(1-i)=2^{283\frac{1}{2}}\cdot 2^{-\frac{1}{2}}(1-i)$$

$$=\boxed{2^{283}(1-i)}$$

c) Schrijf eerst $-8+8\sqrt{3}i$ in de vorm $r(\cos\varphi+i\sin\varphi)$

$r=|-8+8\sqrt{3}i|=\sqrt{(-8)^2+(8\sqrt{3})^2}=8\sqrt{1^2+(\sqrt{3})^2}=8\sqrt{4}=16$

$\cos\varphi=-\frac{8}{16}=-\frac{1}{2}$, $\sin\varphi=\frac{8\sqrt{3}}{16}=\frac{1}{2}\sqrt{3}$, neem $\varphi=\frac{2}{3}\pi$

Dus $-8+8\sqrt{3}i=16\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)$

De oplossingen van $z^8=-8+8\sqrt{3}i$ zijn dus

$$z_k=\sqrt[8]{16}\left(\cos\left(\frac{2\pi/3}{8}+\frac{2k\pi}{8}\right)+i\sin\left(\frac{2\pi/3}{8}+\frac{2k\pi}{8}\right)\right)$$

$$=\boxed{\sqrt{2}\left(\cos\left(\frac{\pi}{12}+\frac{k\pi}{4}\right)+i\sin\left(\frac{\pi}{12}+\frac{k\pi}{4}\right)\right)}$$ $k=0,1,2,3,4,5,6,7$

3 a) Schrijf eerst -4 in de vorm $r(\cos\varphi + i\sin\varphi)$

$-4 = 4(\cos\pi + i\sin\pi)$

$e^{3z} = -4 \Leftrightarrow 3z = \ln 4 + i(\pi + 2k\pi) \quad (k \in \mathbb{Z})$

$\Leftrightarrow \boxed{z = \frac{1}{3}\ln 4 + i\left(\frac{\pi}{3} + \frac{2k\pi}{3}\right) \quad (k \in \mathbb{Z})}$

4 a) $\displaystyle\sum_{n\geq 1}^{\infty} \frac{5\times 3^n - 6\times 2^n}{4^n} = 5\sum_{n\geq 1}^{\infty}\left(\frac{3}{4}\right)^n - 6\sum_{n\geq 1}^{\infty}\left(\frac{2}{4}\right)^n$

$\displaystyle = 5 \times \frac{3/4}{1-3/4} - 6\cdot\frac{1/2}{1-1/2} = 5\,\frac{3/4}{1/4} - 6\,\frac{1/2}{1/2} = 5\times 3 - 6 = \boxed{9}$

opm $\displaystyle\sum_{n=p}^{\infty} r^n = \frac{r^p}{1-r}$, dus $\displaystyle\sum_{n\geq 1}^{\infty} r^n = \frac{r}{1-r}$ voor $-1<r<1$.

b) We gebruiken het quotiëntkenmerk

$\displaystyle\sum_{n\geq 1}^{\infty} a_n$ convergeert als $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$,

en divergeert als $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$.

Pas dit toe met $a_n = \dfrac{10^n}{\sqrt[3]{n!}}$

$\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{10^{n+1}/\sqrt[3]{(n+1)!}}{10^n/\sqrt[3]{n!}} = \lim_{n\to\infty}\frac{10^{n+1}}{10^n}\cdot\frac{\sqrt[3]{n!}}{\sqrt[3]{(n+1)!}}$

$\displaystyle = \lim_{n\to\infty} 10\sqrt[3]{\frac{n!}{(n+1)!}} = \lim_{n\to\infty}\sqrt[3]{\frac{1.2\ldots n}{1.2\ldots n(n+1)}} =$

$\displaystyle = \lim_{n\to\infty} 10\sqrt[3]{\frac{1}{n+1}} = 0 < 1$

Dus $\displaystyle\sum_{n\geq 1}^{\infty}\frac{10^n}{\sqrt[3]{n!}}$ $\boxed{\text{convergeert}}$