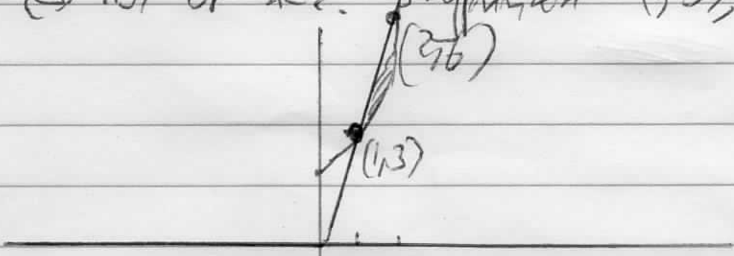
UITWERKING 2E HERKANSING CONTINUE WISKUNDE 2,
14-7-2021

(1) a) Snijpunten: $f(x)=g(x) \Leftrightarrow 3x = x^2+2 \Leftrightarrow x^2-3x+2=0 \Leftrightarrow (x-1)(x-2)=0$
$\Leftrightarrow x=1$ of $x=2$. Snijpunten $(1,3),(2,6)$

Schets gebied:



Oppervlakte $\int_1^2 (f(x)-g(x))\,dx = \int_1^2 (3x-x^2-2)\,dx = \left[\frac{3}{2}x^2-\frac{1}{3}x^3-2x\right]_1^2$

$= \left(\frac{3}{2}\times 4-\frac{1}{3}\times 8-4\right)-\left(\frac{3}{2}-\frac{1}{3}-2\right) = \left(6-\frac{8}{3}-4\right)-\left(\frac{9}{6}-\frac{2}{6}-\frac{12}{6}\right)$

$= -\frac{2}{3}-\left(-\frac{5}{6}\right) = \boxed{\frac{1}{6}}$

b) $\int (3x+6)\sin 3x\,dx = (3x+6)\cdot\left(-\frac{1}{3}\cos 3x\right) - \int \left(-\frac{1}{3}\cos 3x\right)\cdot 3\,dx$

$f(x) = 3x+6$
$g'(x) = \sin 3x$
$g(x) = -\frac{1}{3}\cos 3x$

$= -(x+2)\cos 3x + \int \cos 3x\,dx$

$= \boxed{-(x+2)\cos 3x + \frac{1}{3}\sin 3x + C}$

c) $\int \left(x^5+\frac{1}{3}x+1\right)e^{-x^6-x^2-6x}\,dx = \int e^u \cdot -\frac{1}{6}\,du = -\frac{1}{6}e^u + C$

$u = -x^6-x^2-6x$
$du = (-6x^5-2x-6)\,dx$
$\left(x^5+\frac{1}{3}x+1\right)dx = -\frac{1}{6}du$

$= \boxed{-\frac{1}{6}e^{-x^6-x^2-6x}+C}$

$\int_0^\infty \left(x^5+\frac{1}{3}x+1\right)e^{-x^6-x^2-6x}\,dx = \lim_{B\to\infty} \int_0^B \left(x^5+\frac{1}{3}x+1\right)e^{-x^6-x^2-6x}\,dx$

$= \lim_{B\to\infty} \left[-\frac{1}{6}e^{-x^6-x^2-6x}\right]_0^B = \lim_{B\to\infty} -\frac{1}{6}e^{-B^6-B^2-6B} + \frac{1}{6} = \boxed{\frac{1}{6}}$

(2) a) $\frac{\partial f}{\partial x} = 3x^2 - 12 + y^2$, $\frac{\partial f}{\partial y} = 2(x-1)y$

stationaire punten: $3x^2 - 12 + y^2 = 0$, $2(x-1)y = 0$
$\Leftrightarrow x-1=0$ of $y=0$

$x=1$ invullen in de eerste vergelijking geeft $-9+y^2=0 \Leftrightarrow y=\pm 3$
Dit geeft de stationaire punten $(1,3)$, $(1,-3)$

$y=0$ invullen in de eerste vergelijking geeft $3x^2-12=0$.
$\Leftrightarrow x^2-4=0 \Leftrightarrow x=\pm 2$. Dit geeft de stationaire punten
$(2,0)$, $(-2,0)$

b) $\frac{\partial^2 f}{\partial x^2} = 6x = A$, $\frac{\partial^2 f}{\partial y \partial x} = 2y = B$, $\frac{\partial^2 f}{\partial y^2} = 2(x-1) = C$ $H = AC - B^2$

	A	B	C	H	
(2,0)	12	0	2	24	minimum
(-2,0)	-12	0	-6	72	maximum
(1,3)	6	6	0	-36	zadelpunt
(1,-3)	6	-6	0	-36	zadelpunt

$f(x,0) = x^3 - 12x$ $\lim_{x\to\infty} f(x,0) = \infty$, $\lim_{x\to-\infty} f(x,0) = -\infty$

Dus het maximum en minimum zijn relatief

(3) a) $|z^4/w^6| = |z|^4/|w|^6$. $z = 4+2i \Rightarrow |z| = \sqrt{4^2+2^2} = \sqrt{20}$
$w = 1-2i \Rightarrow |w| = \sqrt{1^2+(-2)^2} = \sqrt{5}$

$|z^4/w^6| = (\sqrt{20})^4/(\sqrt{5})^6 = 20^2/5^3 = \frac{400}{125} = \boxed{\frac{16}{5}}$

b) $2iz^2 + (6-8i)z - 24 = 0 \Leftrightarrow z^2 + \frac{6-8i}{2i}z - \frac{24}{2i} = 0$ $\boxed{30\bar{z}}$

$\Leftrightarrow z^2 + \frac{(6-8i)i}{2i^2}z - \frac{24i}{2i^2} = 0 \Leftrightarrow z^2 + \frac{8+6i}{-2}z - \frac{24i}{-2} = 0 \Leftrightarrow$

$\Leftrightarrow z^2 + (4+3i)z + 12i = 0$

We moeten z_1, z_2 zoeken met $z_1 + z_2 = 4+3i$, $z_1 \cdot z_2 = 12i$

$z_1 = 4$, $z_2 = 3i$ voldoen

$\Leftrightarrow (z+4)(z+3i) = 0 \Leftrightarrow$ $\boxed{z = -4,\ z = -3i}$

c) Schrijf $-4096 - 4096i$ in de vorm $r(\cos\varphi + i\sin\varphi)$

$r = \sqrt{(-4096)^2 + (-4096)^2} = 4096\sqrt{1^2+1^2} = 4096\sqrt{2} = 2^{12\frac{1}{2}}$

$\cos\varphi = \frac{-4096}{4096\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$, $\sin\varphi = \frac{-4096}{4096\sqrt{2}} = -\frac{1}{2}\sqrt{2}$

We mogen $\varphi = -\frac{3}{4}\pi$ nemen $-4096 - 4096i = 2^{\frac{25}{2}}(\cos -\frac{3}{4}\pi + i\sin -\frac{3}{4}\pi)$

Oplossingen van $z^6 = -4096 - 4096i$:

$z_k = \sqrt[6]{2^{\frac{25}{2}}}\left(\cos\left(-\frac{3}{24}\pi + \frac{2k\pi}{6}\right) + i\sin\left(-\frac{3}{24}\pi + \frac{2k\pi}{6}\right)\right)$

$= \boxed{2^{\frac{25}{12}}\left(\cos\left(-\frac{1}{8}\pi + \frac{k\pi}{3}\right) + i\sin\left(-\frac{1}{8}\pi + \frac{k\pi}{3}\right)\right) \quad k = 0,1,2,3,4,5}$

d) $(1+i)e^z = 1 \Leftrightarrow e^z = \frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{1^2+1^2} = \frac{1}{2} - \frac{i}{2}$

Schrijf $\frac{1}{2} - \frac{1}{2}i$ in de vorm $r(\cos\varphi + i\sin\varphi)$

$r = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}$ $\cos\varphi = \frac{1/2}{1/2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$

$\sin\varphi = \frac{-1/2}{\frac{1}{2}\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$. We kunnen $\varphi = -\frac{1}{4}\pi$ nemen

$(1+i)e^z = 1 \Leftrightarrow e^z = \frac{1}{2} - \frac{1}{2}i = \frac{1}{2}\sqrt{2}(\cos(-\frac{1}{4}\pi) - i\sin(-\frac{1}{4}\pi))$

$\Leftrightarrow z = \boxed{\ln\frac{1}{2}\sqrt{2} + i(-\frac{1}{4}\pi + 2k\pi)}$ $(k \in \mathbb{Z})$

(4) a) $\sum_{n=2}^{\infty} \frac{5\times 4^n-(-2)^n}{5^n} = 5\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^n - \sum_{n=2}^{\infty}\frac{(-2)^n}{5^n} = 5\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^n - \sum_{n=2}^{\infty}\left(\frac{-2}{5}\right)^n$

$= 5\,\frac{(4/5)^2}{1-\frac{4}{5}} - \frac{(-2/5)^2}{1-\left(-\frac{2}{5}\right)} = 5\,\frac{(4/5)^2}{1/5} - \frac{(-2/5)^2}{7/5}$

$= 5^2\cdot\frac{4^2}{5^2} - \frac{4/5^2}{7/5} = 4^2 - \frac{4}{25}\times\frac{5}{7} = 4^2 - \frac{4}{5}\cdot\frac{1}{7} = 16 - \frac{4}{35}$

$= \boxed{15\,\tfrac{31}{35}}$

b) $2n^2+\sqrt{n} \approx 2n^2$, $3n^3+\sqrt[3]{n} \approx 3n^3$

dus $\frac{2n^2+\sqrt{n}}{3n^3+\sqrt[3]{n}} \approx \frac{2n^2}{3n^3} \approx \frac{2}{3}\cdot\frac{1}{n}$

We vergelijken $a_n = \frac{2n^2+\sqrt{n}}{3n^3+\sqrt[3]{n}}$ met $b_n = \frac{1}{n}$

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{2n^2+\sqrt{n}}{3n^3+\sqrt[3]{n}}\cdot\frac{1}{1/n} = \lim_{n\to\infty}\frac{(2n^2+\sqrt{n})n}{(3n^3+\sqrt[3]{n})}$

$= \lim_{n\to\infty}\frac{2n^3+n^{3/2}}{3n^3+n^{1/3}} = \lim_{n\to\infty}\frac{2+n^{-3/2}}{3+n^{-8/3}} = \frac{2}{3} > 0$

De reeks $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n}$ is divergent.

Dus $\sum_{n=1}^{\infty}\frac{2n^2+\sqrt{n}}{3n^3+\sqrt[3]{n}}$ is $\boxed{\text{divergent}}$