COUPLING WITH RANDOM DIGITS

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PROBLEM:

Draw 100 digits randomly and independently from $\{1, 2, \ldots, 9, 0\}$. Consider two players who each do the following:

1. Randomly choose one of the first 10 digits.
2. Move forward as many digits as the number that is hit (move forward 10 digits when a 0 is hit).
3. Repeat.
4. Stop when the next move goes beyond digit 100.
5. Record the last digit that is hit.

It turns out that the probability that the two players record the *same* last digit is approximately 0.974.

QUESTIONS:

1. Why is this probability so close to 1?
2. What if N digits are drawn randomly instead of 100 digits?
3. What if there are M players?

GOAL:

One goal of the project is to find a formula for the probability that M players record the same last digit before moving beyond digit N. Another goal is to develop a general theory for the description of such games. This leads to *Coupling Theory*, which is a powerful method in probability theory through which random variables can be compared with each other.