Asymptotic

Conciliation of ‘Bayesian’ & ‘Pointwise’ Q State Estimation

aka tomography

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My Q philosophy:
http://www.math.leidenuniv.nl/~gill/waveparticle.html

The Arena:

- Suppose N moderately large : $1/N \ll 1$, $0 \ll 1/\sqrt{N}$
- Given : N copies of $\rho(\vec{\theta}) = \frac{1}{2}(\mathbb{1} + \vec{\theta} \cdot \vec{\sigma}), \quad \vec{\theta} \in \Theta$
  -- what is $\vec{\theta}$? (measurement + dataprocessing)
- separate / adaptive / LOCC / separable / collective ?
  # of components / measurements ? projective / POVM ? ...
- prior 'knowledge' ? (pure/mixed, 2D/3D : $\vec{\theta} \in \Theta$)
- prior distribution ?? figure of merit ??

The Tools:

(Q) Statistics

- Cramer-Rao ->
  - Holevo (Q CR)
  - van Trees (B CR)

(Q) Probability

- (Q) LLN, (Q) CLT
- Taylor (propagation of errors; d-method)

Analysis

} Q LAN (not in this talk)

INSPIRATION, COLLABORATION:

- Ole Barndorff-Nielsen, Peter Jupp [senior maths]

- Luis Artiles, Manuel Ballester, Madalin Guta, Jonas Kahn phd thesis! <– www [young maths]

- Emili Bagan, Alex Monras, Ramon Muñoz-Tapia, Oriol Romero-Isart [Catalans]

- Keiji Matsumoto, Masahito Hayashi [Japanese]

- Yuen & Lax, Helstrom ; Belavkin, Holevo ; Nagaoka, Fujiwara; Massar & Popescu ... [heroes]

- [d'Ariano c.s. , Buzek, Hradil, ...] [unclassified!]

THE PAPERS:

- RD Gill (2005)
  Asymptotic information bounds in quantum statistics
  Under revision for Annals of Statistics
  math.ST/0512443

- E Bagan, MA Ballester, RD Gill, A Monras & R Muñoz-Tapia (2006)
  Optimal full estimation of qubit mixed states
  Phys Rev A **73** 032301(18)
  quant-ph/0510158

- E Bagan, MA Ballester, RD Gill, R Muñoz-Tapia, O Romero-Isart (2006)
  Separable measurement estimation of density matrices and its fidelity gap with collective protocols
  Phys Rev Lett **97** 130501(4)
  quant-ph/0512177

THE FUTURE (QLAN) :

- MI Guta & J Kahn (2006)
  Local asymptotic normality for qubit states
  Phys Rev A **73** 05218

- MI Guta, B Janssens & J Kahn (2006)
  Optimal full estimation of qubit mixed states
  quant-ph/0608074

- MI Guta & A Jencova (2006)
  Local asymptotic normality in quantum statistics
  quant-ph/0606213

Other References:

- RD Gill & BY Levit (1995)
  Applications of the Van Trees inequality:
  a Bayesian Cramér-Rao bound Bernoulli **1** 59-79
  http://www.math.uu.nl/people/gill/Preprints/van_trees.pdf

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- M Hayashi & K Matsumoto (2004) quant-ph/0411073
  Asymptotic performance of state estimation in quantum two level system

- M Hayashi (2003; 2006) quant-ph/0608198
  Quantum estimation and quantum central limit theorem

Main theorem (Gill, 2005) :

Asympt. quadratic Bayes' risk for est'm of $\psi(\theta)$ with weight matrix $\widetilde{G}(\theta)$ and prior $\pi(\theta)$ is no smaller than the prior mean pointwise Holevo bound with weight matrix $G = \psi'\widetilde{G}\psi'^{\top}$, i.e. :

$$\liminf_{N\to\infty} N R^{(N)}(\pi) \;\geq\; \mathbb{E}_\pi \mathcal{C}_G$$

where

$$R^{(N)}(\pi) \;=\; \mathbb{E}_\pi\, \mathbb{E}_\theta\, \Bigl(\widehat{\psi}^{(N)} - \psi(\theta)\Bigr)^{\top} \widetilde{G}(\theta)\, \Bigl(\widehat{\psi}^{(N)} - \psi(\theta)\Bigr)$$

$$\mathcal{C}_G(\theta) \;=\; \inf_{\substack{\text{s.a. } \vec{X},\, Z\,;\,\text{real } V \;:\; V \,\geq\, Z\,, \\ \operatorname{trace} \frac{\partial}{\partial\theta_i}\rho(\theta) X_j \;=\; \delta_{ij}\,, \\ \operatorname{trace} \rho(\theta) X_i X_j \;=\; Z_{ij}}} \operatorname{trace} G(\theta) V$$

(Holevo bound)

(this part live on real blackboard)

Sketch of proof: ...

(a math talk without a proof is like a movie without a love scene)

(this part live on real blackboard)

Examples:

completely unknown mixed qubit fidelity loss
3D and 2D cases
(2D case insoluble without these methods)

completely unknown pure qudit

ANY covariant exhaustive povm + wise data processing
is asymptotically optimal, eg,

uniform random independent
von Neumann measurements + MLE or Bayes

CONCLUSIONS:

wise combination of complementary ideas
from Bayesian and pointwise approaches
is needed to gain theoretical insight
and solve practical problems

My point of view:
I am a rabid pragmatist
ie fanatically opposed BOTH to rabid Bayesians
AND to rabid frequentists

An often profitable approach is to study the frequentist properties
of Bayesian methods, in order to tune priors and loss functions
to attain sensible frequentist properties