# Statistical thinking in
# "the experiment of the century"

- "If you need statistics, you did the wrong experiment" – Ernest Rutherford

Ernest Rutherford, 1st Baron Rutherford of Nelson, OM, FRS (30 August 1871 – 19 October 1937) was a New Zealand-born British physicist who became known as the father of nuclear physics. Encyclopædia Britannica considers him to be the greatest experimentalist since Michael Faraday

# Towards a definitive and successful Bell-type experiment





Bell (1964): <u>quantum mechanics</u> (QM) is incompatible
<u>local hidden variables</u> aka <u>local realism</u>
(**locality + realism**)
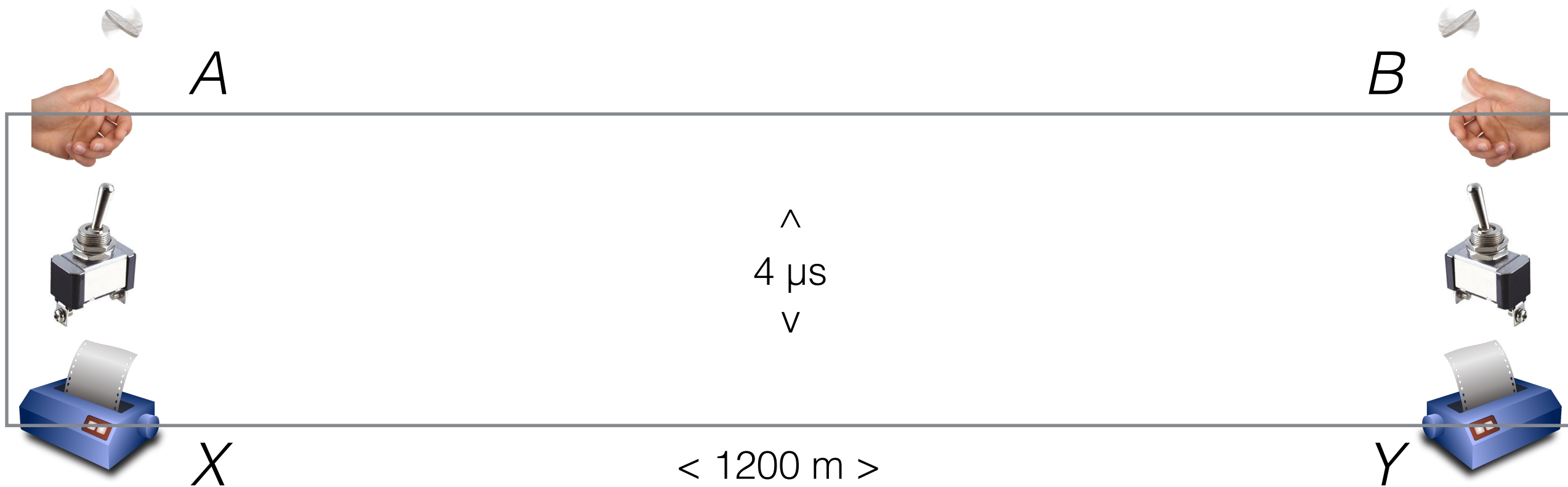(provided we assume **freedom / no conspiracy**)

Experimental evidence: …, Aspect (1982), Weihs (1998), Giustina (2013),
Christensen (2013) …  but not good enough!

Why? technological limitations → loopholes → statistical issues familiar in
epidemiology!

time (4 μs)

space (4000 ft)

A

B

∧
4 μs
∨

X

< 1200 m >

Y

setting A → outcome X

outcome Y ← setting B

Alice

Bob

speed of light = 1 foot per nanosecond

# What is this QM?

- QM models the probability distribution of measurement outcomes, it does not tell us what actually happens

- Measurement outcomes - orthogonal subspaces of complex Hilbert space

- Quantum states - unit vectors in Hilbert space

- Composite systems, measurements, modelled by tensor product

- The probability of a particular outcome = squared length of projection of state vector into corresponding subspace

Pythagoras!

# Bell's inequality, Bell's theorem

- <u>Settings</u> $A$, $B$ take values in $\{1, 2\}$

- <u>Counterfactual outcomes</u> $X_{11}$, $X_{12}$, $X_{21}$, $X_{22}$, $Y_{11}$, $Y_{12}$, $Y_{21}$, $Y_{22}$ take values in $\{-1, +1\}$

- <u>Actual outcomes</u> $X = X_{AB}$, $Y = Y_{AB}$

- **<u>Freedom (no conspiracy)</u>**
  $(A, B) \perp\!\!\!\perp (X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{21}, Y_{22})$

# Locality, realism, freedom

- **Realism** = existence of counterfactual outcomes

- **Locality** = Alice's outcomes don't depend on Bob's settings and vice-versa

  - $X_1 := X_{11} = X_{12}$ , $X_2 := X_{21} = X_{22}$

  - $Y_1 := Y_{11} = Y_{21}$ , $Y_2 := Y_{12} = Y_{22}$

  - $X = X_A$, $Y = Y_B$

- **Freedom** = statistical independence of actual settings from counterfactual outcomes

  - $(A, B) \perp\!\!\!\perp (X_1, X_2, Y_1, Y_2)$

# Bell's inequality, Bell's theorem
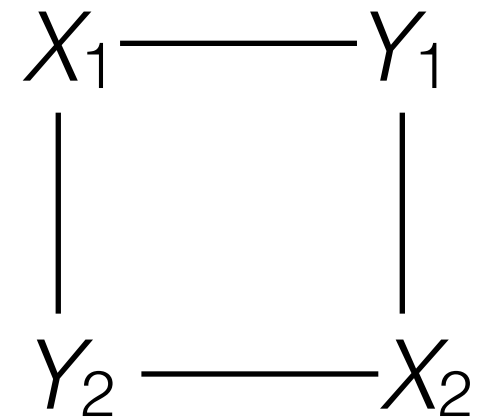
$$X_1 \text{——} Y_1$$
$$| \quad\quad |$$
$$Y_2 \text{——} X_2$$

$$X_1 = Y_2 \quad \& \quad Y_2 = X_2 \quad \& \quad X_2 = Y_1 \quad \Rightarrow \quad X_1 = Y_1$$

$\therefore$ $\boxed{X_1 \neq Y_1 \quad \Rightarrow \quad X_1 \neq Y_2 \quad \text{or} \quad Y_2 \neq X_2 \quad \text{or} \quad X_2 \neq Y_1}$
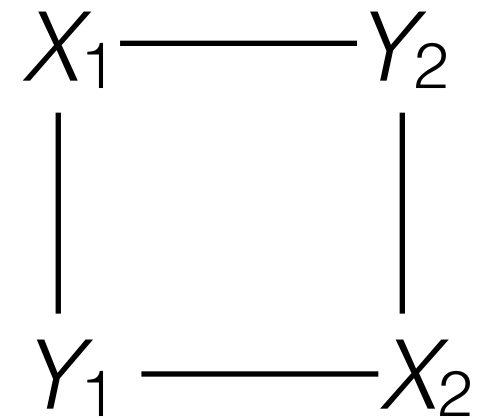
$\therefore$ $P(X_1 \neq Y_1) \leq P(X_1 \neq Y_2) + P(Y_2 \neq X_2) + P(X_2 \neq Y_1)$

$\therefore$ $\boxed{P_{11}(X \neq Y) \leq P_{12}(X \neq Y) + P_{22}(Y \neq X) + P_{21}(X \neq Y)}$

where $P_{ab}(\dots) = P(\dots \mid A = a, B = b)$

NB:(probabilistic) Bell inequality is actually just
a simple corollary of a <u>logical implication</u>

# Bell's inequality, Bell's theorem

$$X_1 \text{——} Y_2$$
$$| \qquad |$$
$$Y_1 \text{——} X_2$$

$P_{11}(X \neq Y) \leq P_{12}(X \neq Y) + P_{22}(Y \neq X) + P_{21}(X \neq Y)$

For instance: $0.75 \leq 0.25 + 0.25 + 0.25$

But QM promises we can get (approx) 0.85, 0.15, 0.15, 0.15

Notice $E_{ab}(XY) = P_{ab}(X = Y) - P_{ab}(X \neq Y) = 1 - 2 P_{ab}(X \neq Y)$

Define $S := E_{12}(XY) + E_{22}(XY) + E_{21}(XY) - E_{11}(XY) =: \rho_{12} + \rho_{22} + \rho_{21} - \rho_{11}$

Equivalently: $S \leq 2$          (CHSH: Clauser-Horne-Shimony-Holt, 1969)

QM promises we can (maximally) get $2\sqrt{2}$          (why not 4 ??)
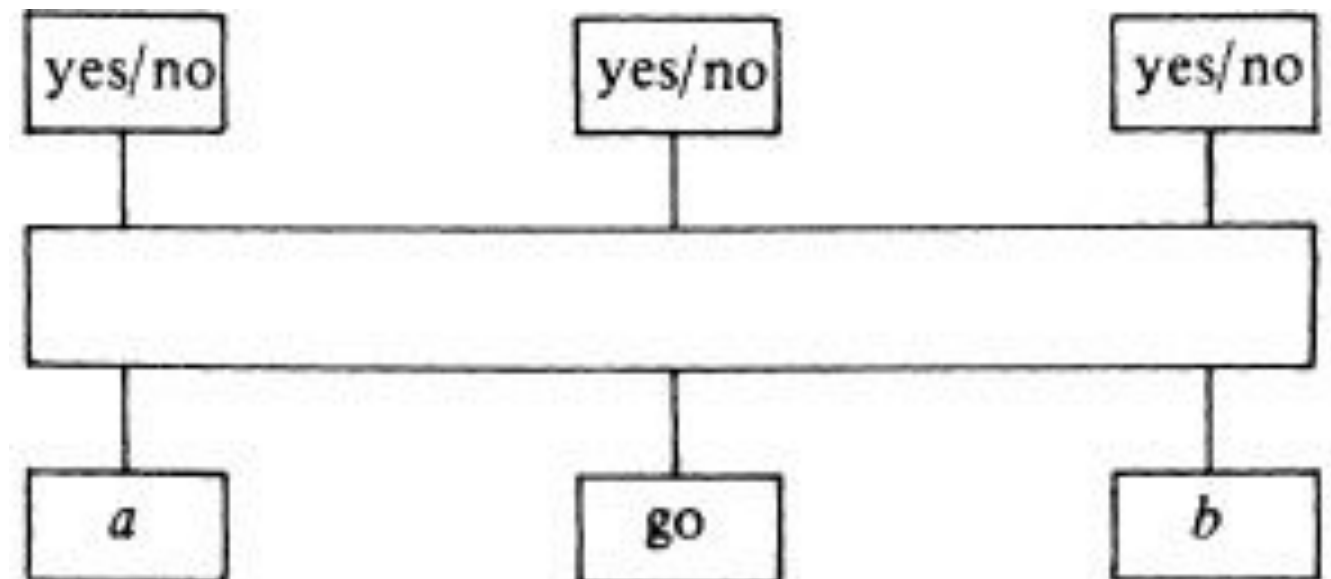
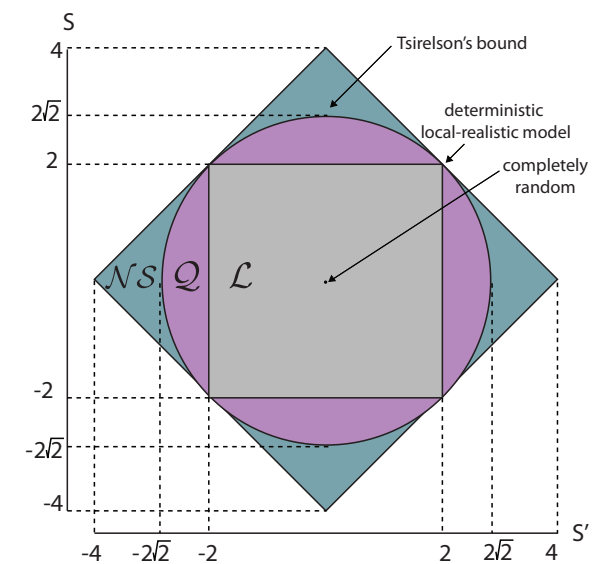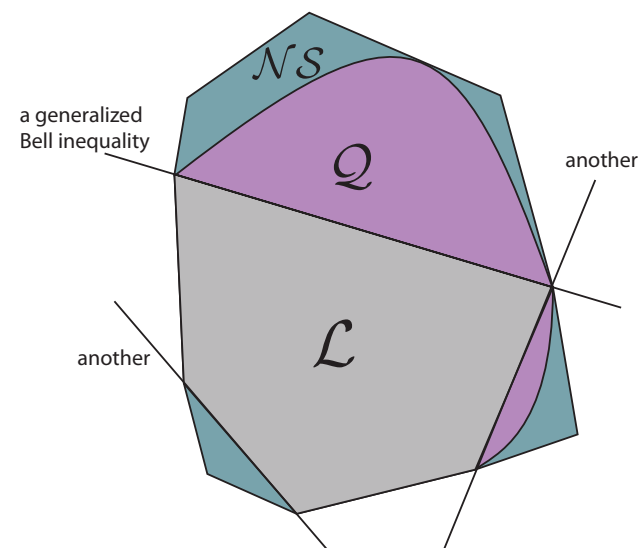Bell, 1964          Tsirelson, 1980          Pawlowski, 2009

John Bell explains: "I cannot say that action at a distance is required in physics. But I cannot say that you can get away with no action at a distance. You cannot separate off what happens in one place with what happens at another"

https://www.youtube.com/watch?v=V8CCfOD1iu8

Bell (1981) Bertlmann's socks and the nature of reality



Les chaussettes de M. Bertlmann et la nature de la réalité

Fondation Hugot juin 17 1980

pink        not pink

yes/no          yes/no          yes/no

a               go               b

# The local polytope, quantum convex body, no-signalling polytope



Definitions:

$$p_{x,y,a,b} = p(x, y \mid a, b) = P(X = x, Y = y \mid A = a, B = b)$$

$$= P(X_a = x, Y_b = y) \quad \text{under locality + realism + freedom}$$

$$\mathbf{p} = ( \, p(x, y \mid a, b)_{x,y = \pm 1; a,b=1,2} \, ) \ \in \ \mathbf{R}^{16}$$

# The local-realism polytope, quantum convex body, no-signalling polytope

$p_{x,y,a,b} = p(x, y \mid a, b) = \mathrm{P}(X = x, Y = y \mid A = a, B = b)$

$\mathbf{p} = (\, p_{x,y,a,b} : x, y = \pm 1; a, b = 1,2 \,) \in \mathbf{R}^{16}$
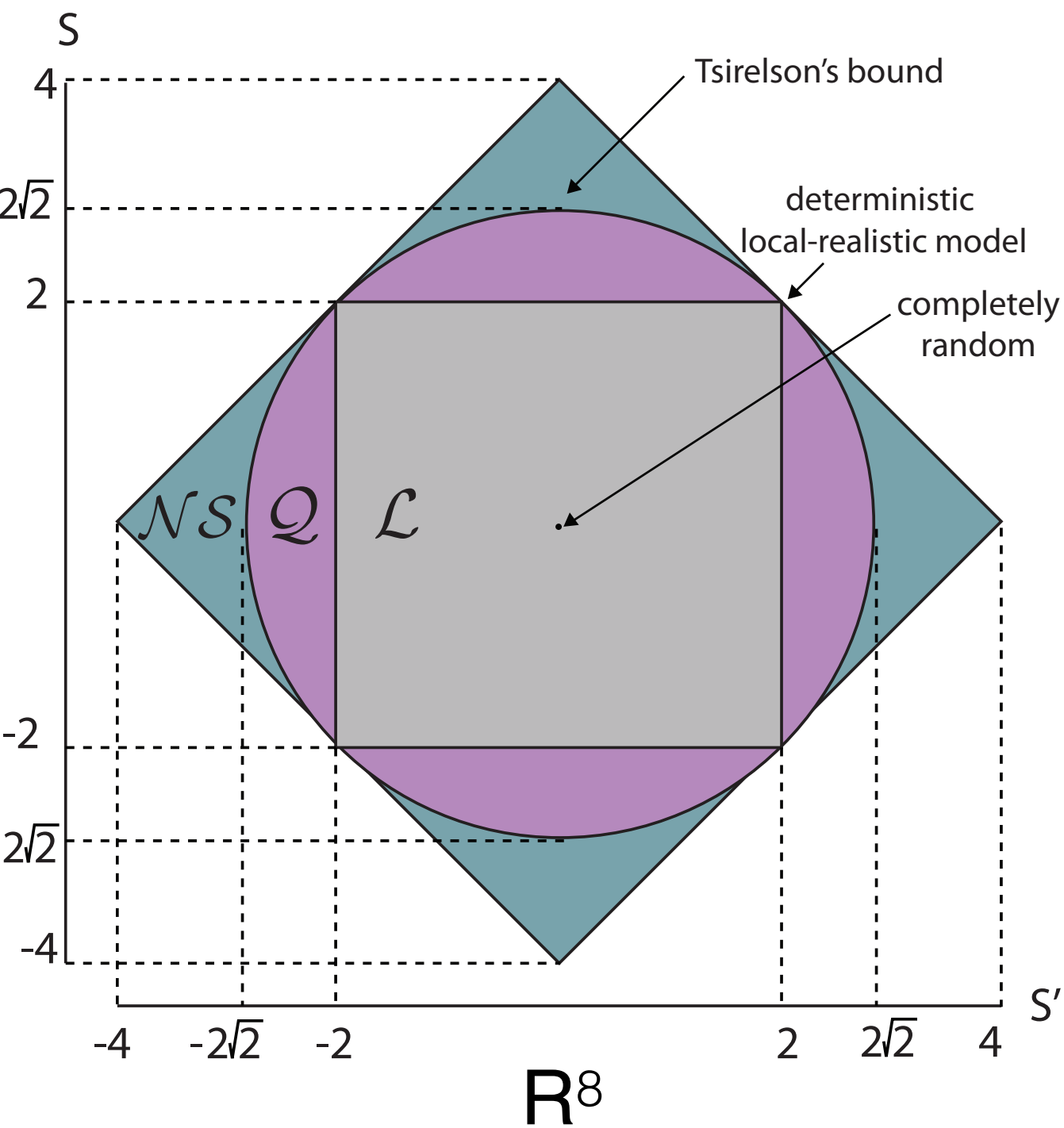
*Nonnegativity* inequalities: $\forall_{x,y,a,b} \; p_{x,y,a,b} \geq 0$

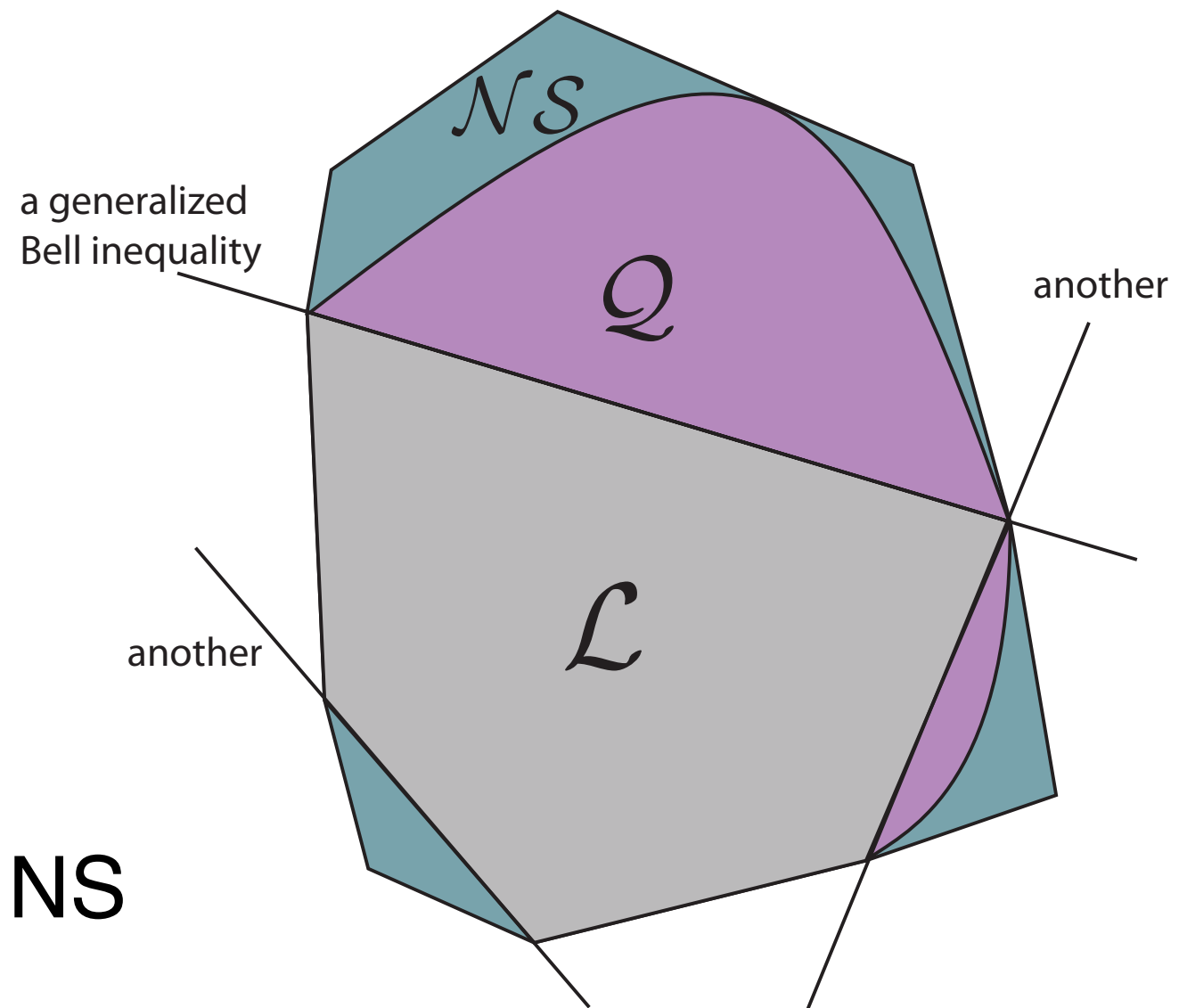*Normalisation* equalities: $\forall_{a,b} \; \sum_{x,y} p_{x,y,a,b} = 1$

*No-signalling* equalities: $\forall_a \; \sum_y p_{x,y,a,b}$ same for all $b$,
$\forall_b \; \sum_x p_{x,y,a,b}$ same for all $a$

NB: *no-signalling* is a property of all decent physical models. Not just LHV theories …

Caricatures of the local-realism, quantum, no-signalling convex bodies

$$L \subsetneq Q \subsetneq NS$$

Generalised Bell inequalities: $p$ parties, $q$ settings, $r$ outcomes
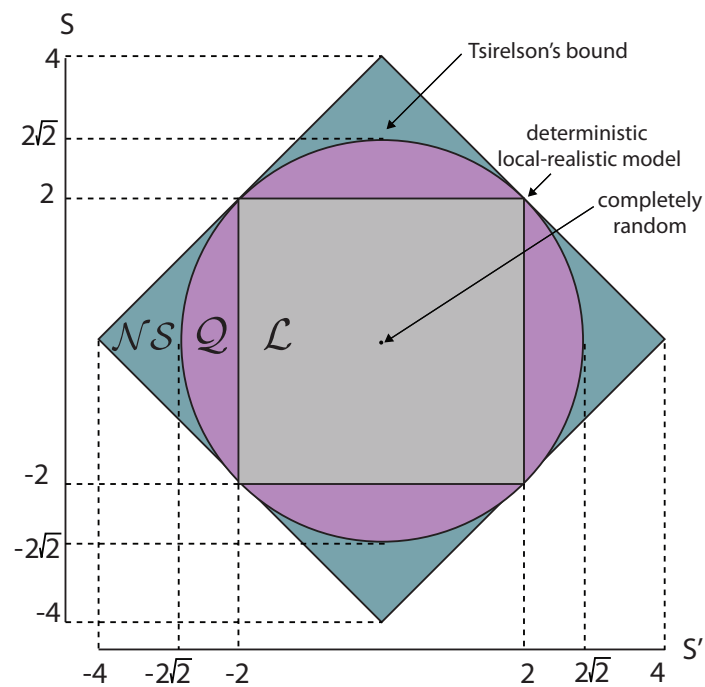Left 2 x 2 x 2, right $p$ x $q$ x $r$

## Local-realism extreme point S = ± 2

| 1 | 0 |
|---|---|
| 0 | 0 |

| 0 | 1 |
|---|---|
| 0 | 0 |

| 1 | 0 |
|---|---|
| 0 | 0 |

| 0 | 1 |
|---|---|
| 0 | 0 |

## Local-realism S = ± 2 nr QM best

| 0,375 | 0,125 |
|---|---|
| 0,125 | 0,375 |

| 0,375 | 0,125 |
|---|---|
| 0,125 | 0,375 |

| 0,375 | 0,125 |
|---|---|
| 0,125 | 0,375 |

| 0,125 | 0,375 |
|---|---|
| 0,375 | 0,125 |

## 2x2x2 case



## Quantum extreme point S = ± 2√2

| 0,43 | 0,07 |
|---|---|
| 0,07 | 0,43 |

| 0,43 | 0,07 |
|---|---|
| 0,07 | 0,43 |

| 0,43 | 0,07 |
|---|---|
| 0,07 | 0,43 |

| 0,07 | 0,43 |
|---|---|
| 0,43 | 0,07 |

## No-signalling extreme point S = ± 4

| 0,5 | 0,0 |
|---|---|
| 0,0 | 0,5 |

| 0,5 | 0,0 |
|---|---|
| 0,0 | 0,5 |

| 0,5 | 0,0 |
|---|---|
| 0,0 | 0,5 |

| 0,0 | 0,5 |
|---|---|
| 0,5 | 0,0 |

# Experiment

- The definitive experiment has now been done

- It was done first, in Delft (and soon after in at least three other places)

# Delft:
# the Hanson diamond group

## Heralded entanglement between solid–state qubits separated by three metres

H. Bernien[1], B. Hensen[1], W. Pfaff[1], G. Koolstra[1], M. S. Blok[1], L. Robledo[1], T. H. Taminiau[1], M. Markham[2], D. J. Twitchen[2], L. Childress[3] & R. Hanson[1]

## Unconditional quantum tele between distant solid-state quantum bits

W. Pfaff,[1*] B. J. Hensen,[1] H. Bernien,[1] S. B. van Dam,[1] M. S. Blok,[1] T. H. Taminiau,[1] M. J. Tiggelman,[1] R. N. Schouten,[1] M. Markham,[2] D. J. Twitchen,[2] R. Hanson[1]†

Realizing robust quantum information transfer between long-lived qubit registers is a key challenge for quantum information science and technology. Here we demonstrate unconditional teleportation of arbitrary quantum states between diamond spin qubits separated by 3 meters. We prepare the teleporter through photon-mediated heralded entanglement between two distant electron spins and subsequently encode the source qubit in a single nuclear spin. By realizing a fully deterministic Bell-state measurement combined with real-time feed-forward, quantum teleportation is achieved upon each attempt with an average state fidelity exceeding the classical limit. These results establish diamond spin qubits as a prime candidate for the realization of quantum networks for quantum communication and network-based quantum computing.

# Delft:
# the Hanson diamond group

**Diamonds**
Hanson's research group produces qubits using electrons in diamonds. 'We use diamonds because 'mini prisons' for electrons are formed in this material whenever a nitrogen atom is located in the position of one of the carbon atoms. The fact that we're able to view these miniature prisons individually makes it possible for us to study and verify an individual electron and even a single atomic nucleus. We're able to set the spin (rotational direction) of these particles in a predetermined state, verify this spin and subsequently read out the data. We do all this in a material that can be used to make chips out of. This is important as many believe that only chip-based systems can be scaled up to a practical technology,' explains Hanson.

**Holy Grail**
Hanson is planning to repeat the experiment this summer over a distance of 1300 metres, with chips located in various buildings on TU Delft's campus. This experiment could be the first that meets the criteria of the 'loophole-free Bell test', and could provide the ultimate evidence to disprove Einstein's rejection of entanglement. Various research groups, including Hanson's, are currently striving to be the first to realise a loophole-free Bell test, which is considered 'Holy Grail' within quantum mechanics.
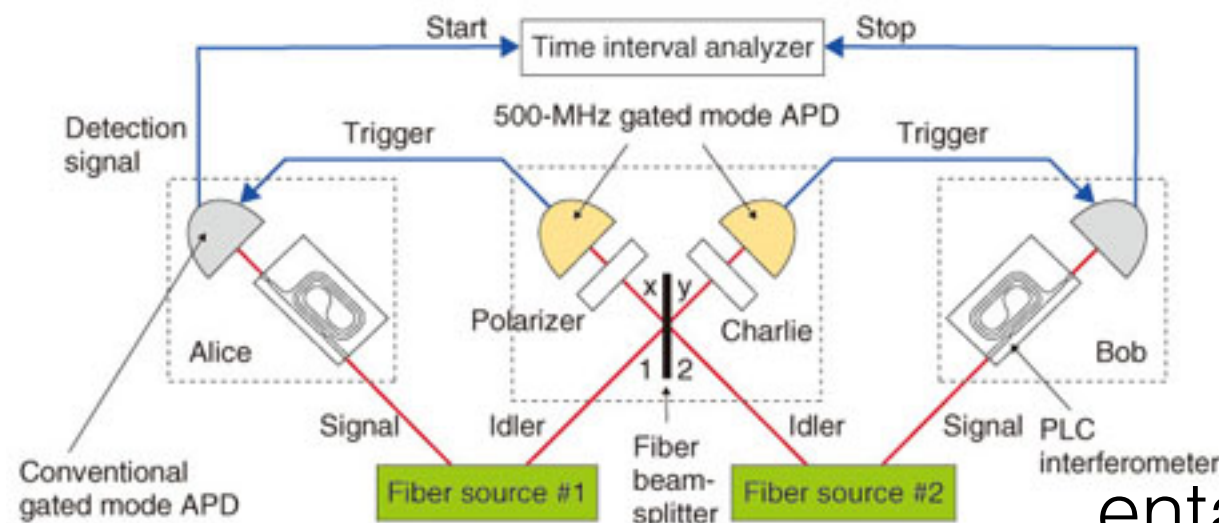
29 May 2014



entanglement swapping

Alice lab

1.3 kilometers

Bob lab
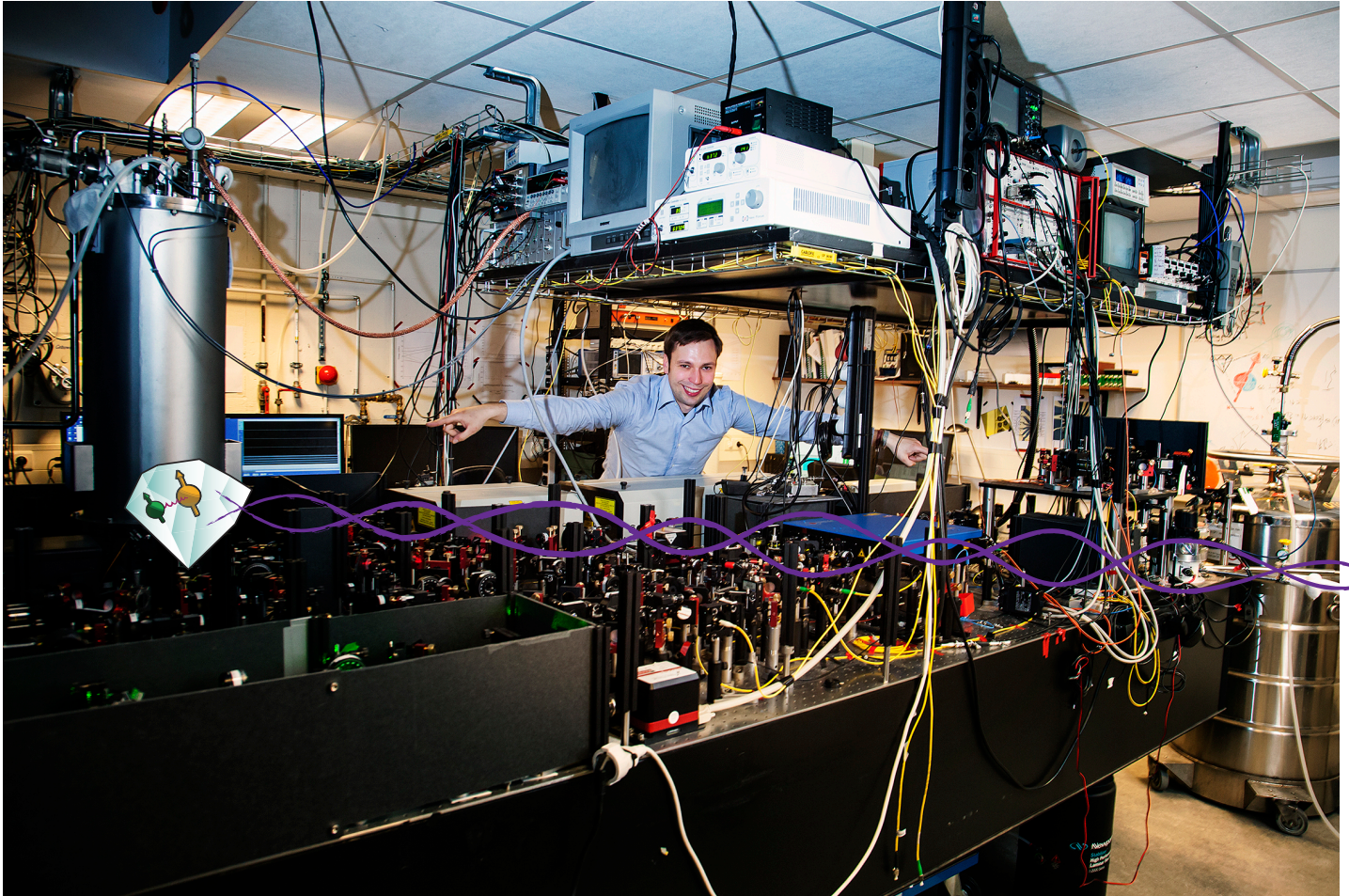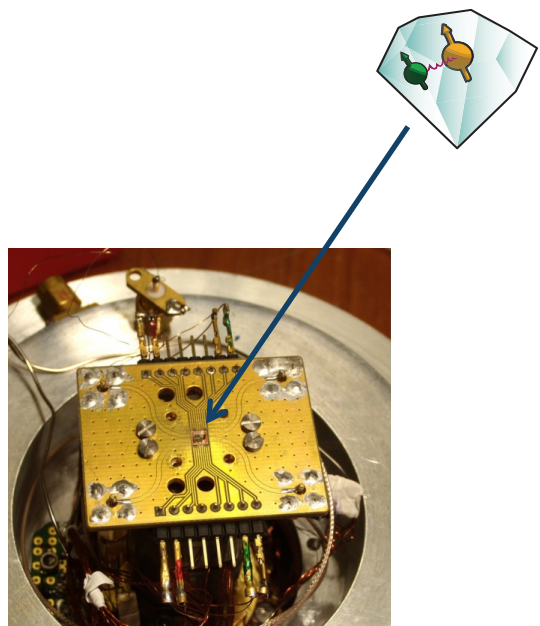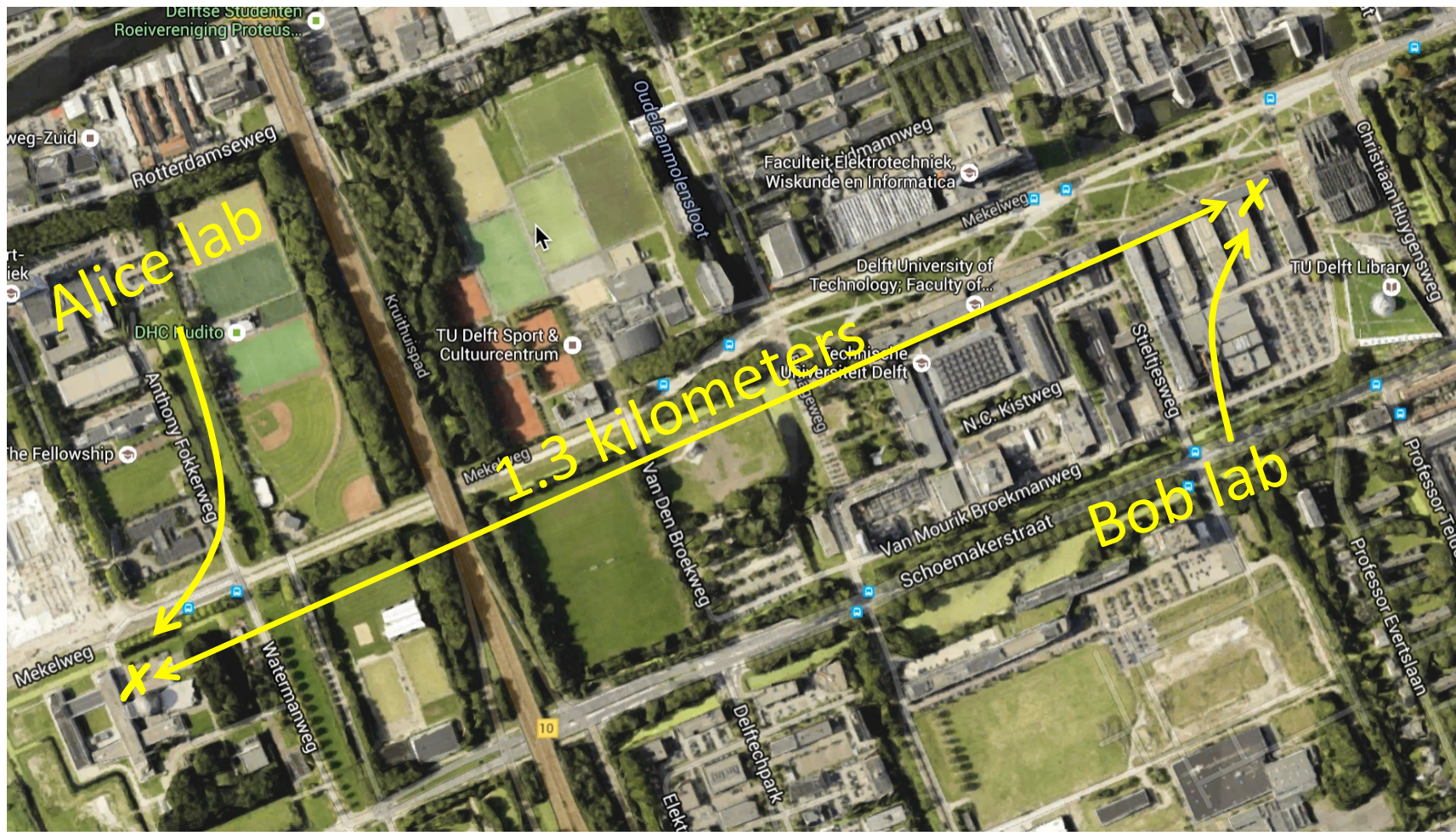
# Statistical issues

- Detection loophole – intention to treat principle!

- Memory loophole – randomisation and martingale theory!

- Time variation – randomisation and martingale theory!

- Statistical significance: based on randomisation of settings, not on an assumed physical model (no "iid" assumption)

- Time variation – turn a <u>bug</u> into a <u>feature</u> by dynamic optimisation of test statistic

- Statistical optimisation – from "$S \leq 2$" to

$$S + \sum_{\text{no-signalling equalities}} c_{\text{n.s.e.}} \, \text{n.s.e.} \leq 2$$

http://rpubs.com/gill1109

# Turning around the randomness: Bell inequalities are logical inequalities

- Suppose $A$, $B$ are independent fair Bernoulli

- Define $I_{ab} = I(A = a, B = b)$

- Condition on values of $X_1$, $X_2$, $Y_1$, $Y_2$ and define
$\delta_{ab} = I(x_a \neq y_b)$

- Observe that
$$E(\delta_{11}\, I_{11} - \delta_{12}\, I_{12} - \delta_{22}\, I_{22} - \delta_{21}\, I_{21}) \leq 0 \quad \text{because}$$
$$x_1 \neq y_1 \;\Rightarrow\; x_1 \neq y_2 \;\text{ or }\; y_2 \neq x_2 \;\text{ or }\; x_2 \neq y_1$$

**This leads to martingale tests: protection against:
time dependence, trends and changes,
opportunistic stopping or skipping**

# Want to know more?

- http://www.slideshare.net/gill1109/epidemiology-meets-quantum-statistics-causality-and-bells-theorem

- http://www.math.leidenuniv.nl/~gill

- Survey paper in *Statistical Science*
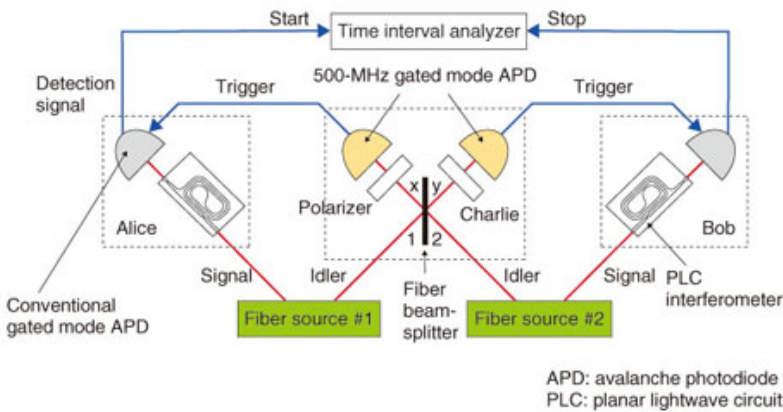
## Statistics, Causality and Bell's Theorem

**Richard D. Gill**

*Abstract.* Bell's [*Physics* **1** (1964) 195–200] theorem is popularly supposed to establish the nonlocality of quantum physics. Violation of Bell's inequality in experiments such as that of Aspect, Dalibard and Roger [*Phys. Rev. Lett.* **49** (1982) 1804–1807] provides empirical proof of nonlocality in the real world. This paper reviews recent work on Bell's theorem, linking it to issues in causality as understood by statisticians. The paper starts with a proof of a strong, finite sample, version of Bell's inequality and thereby also of Bell's theorem, which states that quantum theory is incompatible with the conjunction of three formerly uncontroversial physical principles, here referred to as *locality*, *realism* and *freedom*.

Signal, Idler, Fiber source #1, Fiber beam-splitter, Idler, Signal, PLC interferometer, Fiber source #2

APD: avalanche photodiode
PLC: planar lightwave circuit

# Postscript

I cannot say that action at a distance is required in physics. But I cannot say that you can get away with no action at a distance. You cannot separate off what happens in one place with what happens at another – John Bell

https://www.youtube.com/watch?v=V8CCfOD1iu8

Nature produces chance events (irreducibly chance-like!) which can occur at widely removed spatial locations without anything propagating from point to point along any path joining those locations. … The chance-like character of these effects prevents any possibility of using this form of non locality to communicate, thereby saving from contradiction one of the fundamental principles of relativity theory according to which no communication can travel faster than the speed of light – Nicolas Gisin

Quantum Chance: Nonlocality, Teleportation and Other Quantum Marvels. Springer, 2014