DOES PURE CHANCE EXIST?

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Coincidence and Chance, EURANDOM,
Eindhoven, 11-12-13

DETERMINISM OR CHANCE?

Presocratic philosophers (Empedocles, Democritos, ...)
saw chance as a creative agent, but they were superseded:

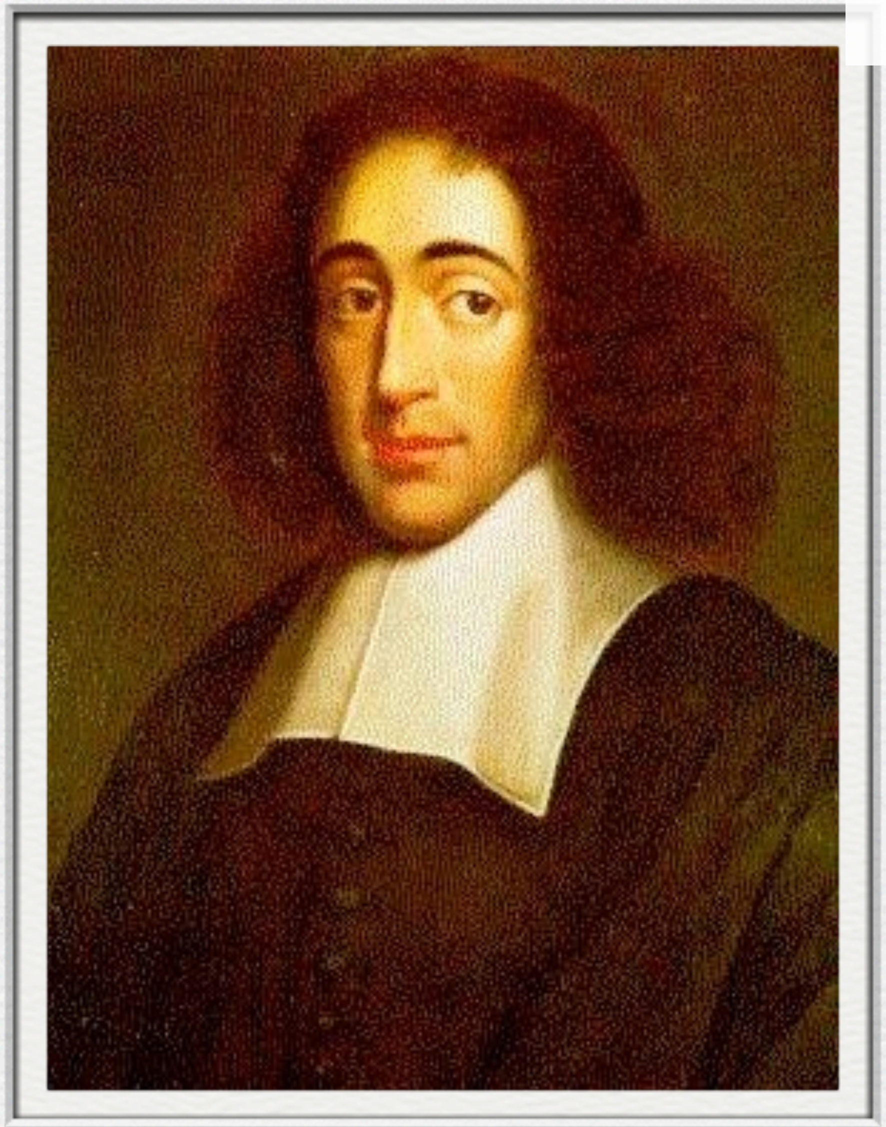
Most major thinkers from -4th till 20th Century *except Newton*
(Plato, Augustine, Aquinas, Spinoza, Leibniz, Laplace ...)
regarded chance as sign of either *corruption* or *ignorance:*
the divine and/or natural order was supposed to be *determined*

Mathematical probability theory created in *vulgar* context of *gambling!*
(Fermat, Pascal, Huygens, 17th Century)

“The first day of Creation wrote what the Day of Judgement will read”

Omar Khayyam
(1048-1131)

“God could not have made things in a different way or in a different order”



Spinoza
(1632-1677)

“One sees then that everything proceeds mathematically - that is, infallibly - in the whole wide world, so that if someone could have sufficient insight into the inner parts of things, and in addition has remembrance and intelligence enough to consider all the circumstances and to take them into account, he would be a prophet and would see the future in the present as in a mirror”

Leibniz
(1646-1716)

THE TWENTIETH CENTURY

Suggestion that 'pure chance' (= indeterminism) might exist

i.e. chance that is not due to lack of computational power and/ or ignorance about state, time-evolution, Laws of Nature, ...

- Genetics, Population Studies, Philosophy (C.S. Peirce)

- The turning point: ***Quantum Mechanics***

Most founders heavily lobbied that QM be indeterministic

QUANTUM MECHANICS

- 1687-1915: Classical physics (Newton - Maxwell - Einstein)
- 1900-1925: Period of crisis (Planck - Einstein - Bohr)
- 1925-1926: Quantum Theory (Heisenberg - Schrödinger)
- 1926-1927: Probability Interpretation (Born - Heisenberg - Dirac)
- 1927-1949: Einstein's critique of QM (versus Born and Bohr):

"*Die Theorie liefert viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, daß er nicht würfelt.*" (Einstein to Born, 1926)


Je gaat het
pas zien als je
het doorhebt.
cruijff
foundation
Johan Cruijff

MATHEMATICAL ARGUMENTS

Two key results **proving** indeterminism (under some assumptions):

- ***Bell's Theorem*** (for deterministic hidden variable theories, 1964)
- ***Free Will Theorem*** (Conway & Kochen, 2006 - 2009)

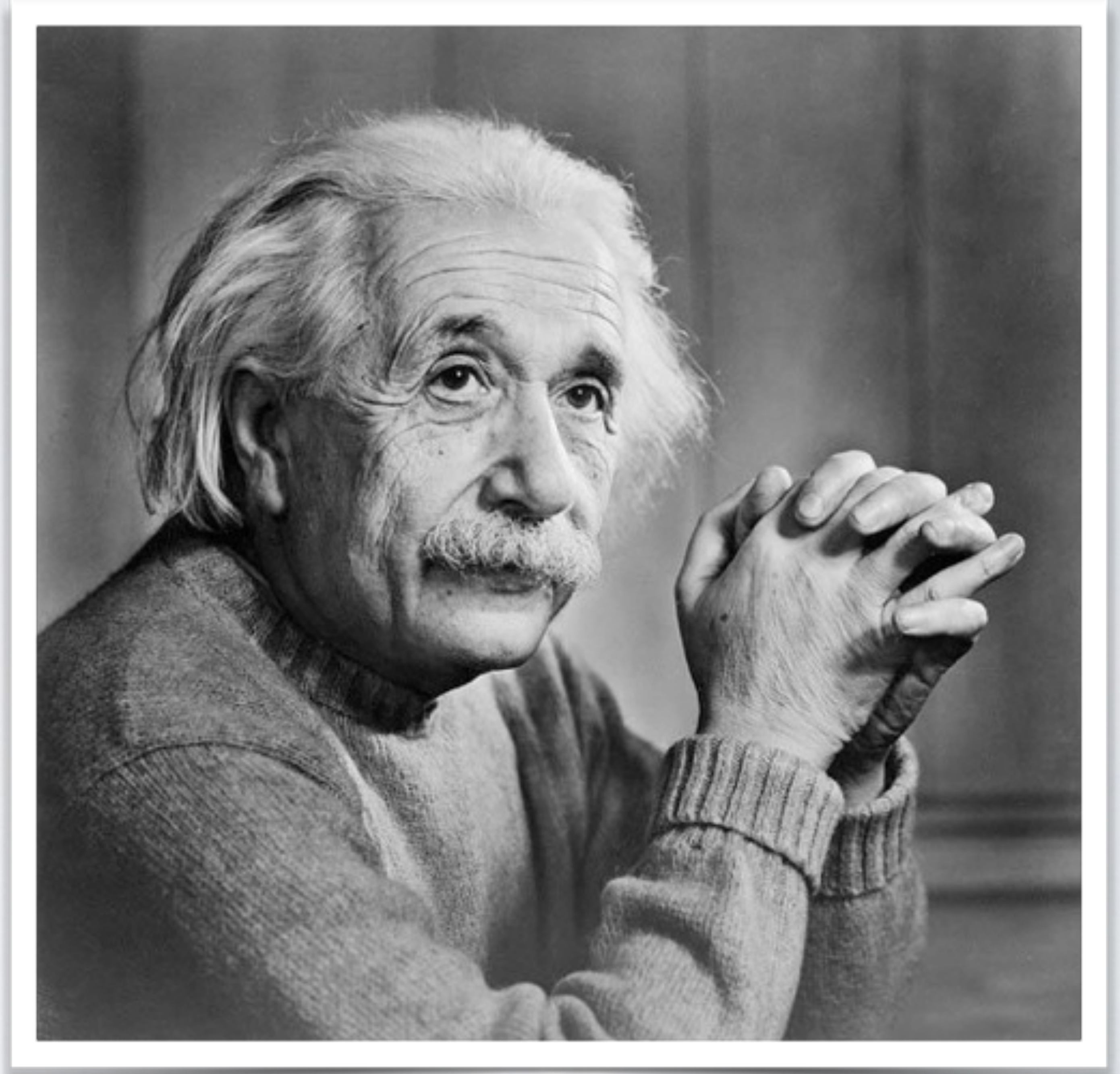
Both originate with von Neumann (1932), breeding two traditions:

- physicists *opposing* his analysis (Einstein, Schrödinger, De Broglie, Bohm, Bell, ...)
- mathematicians *refining* it (Gleason 1958, Kochen-Specker 1967, Conway-Kochen)

Both traditions rely on Einstein-Podolsky-Rosen (EPR) paper:
'Can quantum-mechanical description of physical reality be considered complete?' (1935)

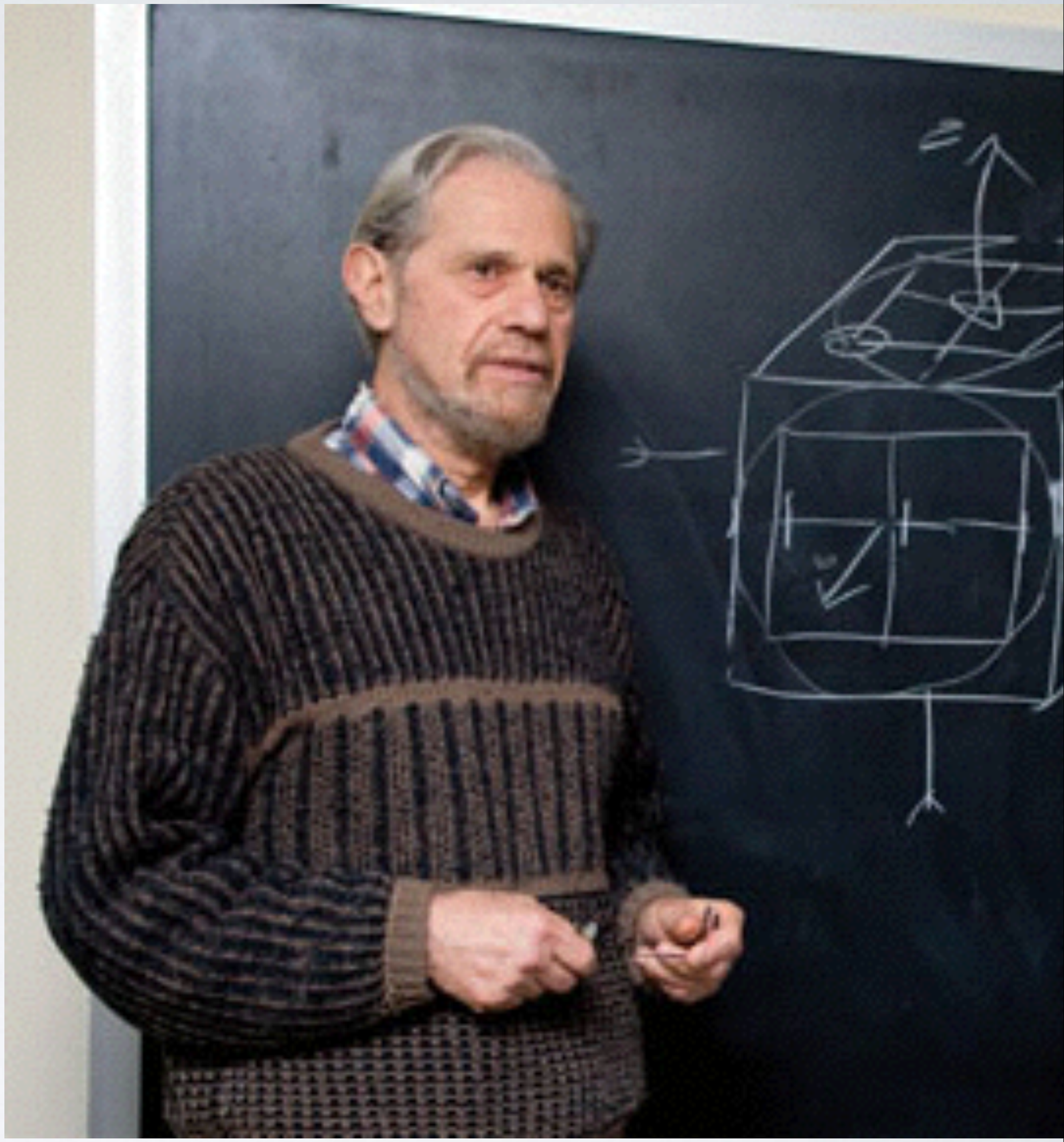
N.B. Rigorous arguments are important e.g. for quantum cryptography!

John von Neumann
(1903-1957)



Albert Einstein
(1879-1955)

John Stuart Bell
(1928-1990)



Simon Kochen
(1934)

SYNOPSIS

Bell's Theorem and Free Will Theorem prove contradiction between:

- ***Determinism*** (of *theories* predicting certain data)
- ***Free Choice*** (of settings of experiment)
- ***Locality*** (impossibility of superluminal signaling)

and

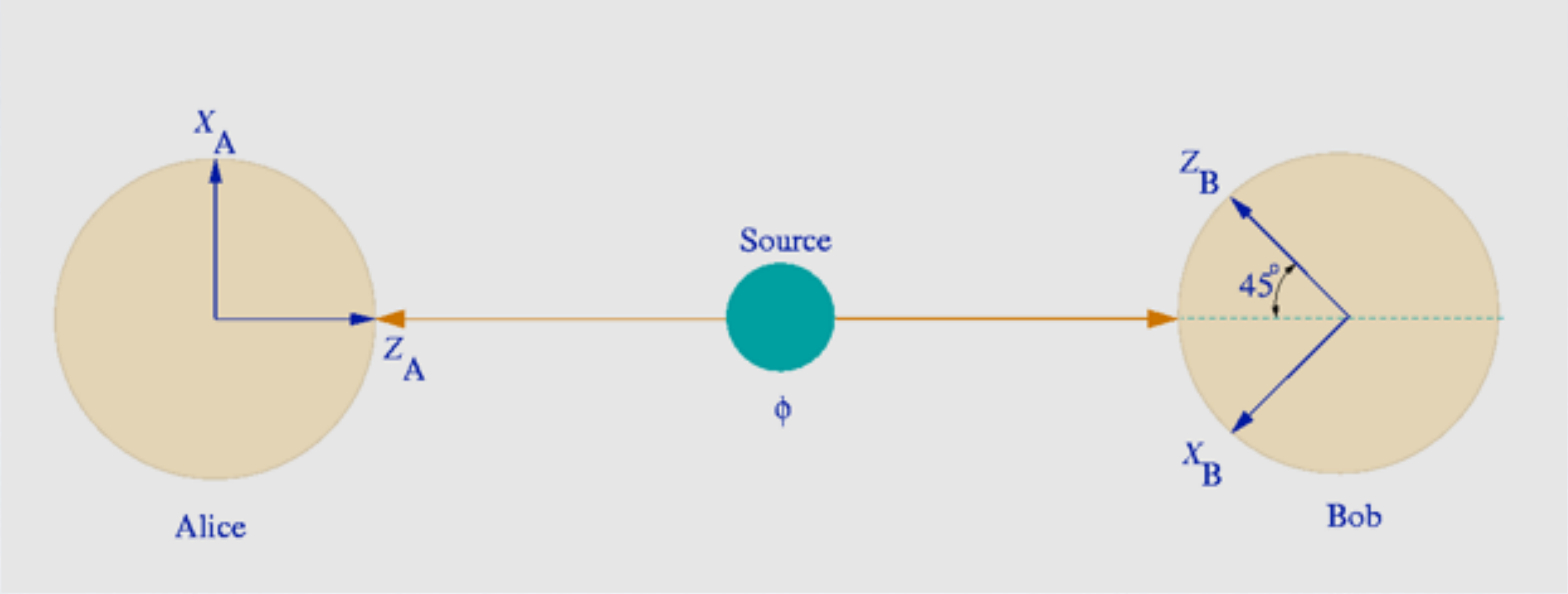
either

- ***Probability theory*** and ***empirical data*** (Bell)

or

- ***Quantum Mechanics of Spin-1*** (Conway-Kochen)

EPR-BOHM EXPERIMENT



- Source atom produces correlated photon pairs
- A (B) = direction of Alice's (Bob's) axis of polarization
- F = 1 (F = 0): Alice's photon does (not) pass through
- G = 1 (G = 0): Bob's photon does (not) pass through

$$P(F \neq G \mid A = a, B = b) = \sin^2(a-b)$$

BELL'S THEOREM (FOR PROBABILISTS)

The following assumptions are contradictory:

- *Determinism*: there exist a state space Ω and functions (F, G, A, B) on Ω such that $x \in \Omega$ determines the settings $a = A(x)$, $b = B(x)$ as well as the outcomes $F(x) \in \{0,1\}$, $G(x) \in \{0,1\}$ of the experiment (so if we knew x ...)

- *Probability Theory*: empirical probabilities $P(F = f, G = g | A = a, B = b)$, $f, g \in \{0,1\}$, are theoretically reproduced as joint conditional probabilities by some *probability* space (Ω, Σ, P) with above *random* variables (F, G, A, B)

- *Free Choice*: $F = F(A, B, Z)$, $G = G(A, B, Z)$ for some random variable Z such that (A, B, Z) are independent with respect to P (condition on P!)

- *Locality*: $F(A, B, Z) = F(A, Z)$ and $G(A, B, Z) = G(B, Z)$

- *Empirical data*: $P(F \neq G \mid A = a, B = b) = \sin^2(a-b)$

PROOF OF BELL'S THEOREM

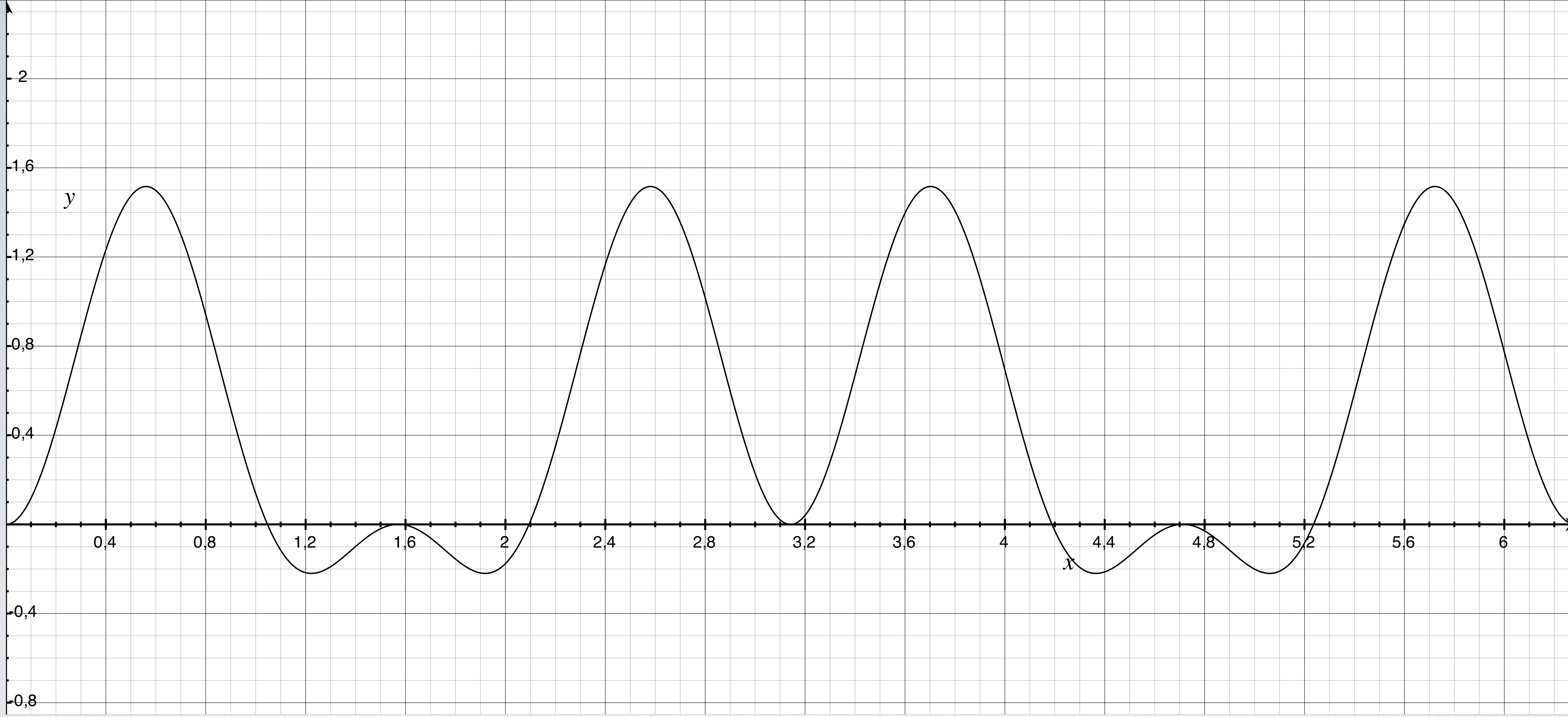
- Let $Z : \Omega \rightarrow \Omega_Z$, $\underline{\Omega} = [0,1] \times [0,1] \times \Omega_Z$, $d\underline{P} = ds \cdot dt \cdot P_Z$
- $\underline{F}_a(s, t, z) = \chi_{[0, P(F = 1, Z = z| A = a))]}(s)$, $\underline{G}_b(s, t, z) = \chi_{[0, P(G = 1, Z = z| B = b))]}(t)$
- Assumptions $\Rightarrow$ $P(F = f, G = g| A = a, B = b) = \underline{P}(\underline{F}_a = f, \underline{G}_b = g)$
- Any four {0, 1}-valued random variables E, F, G, H satisfy

$$P(E \neq G) \leq P(E \neq H) + P(F \neq G) + P(F \neq H)$$

- For suitable angles a, a', b, b', this is violated by $E = \underline{F}_a, \dots, H = \underline{G}_{b'}$

$$\underline{P}(\underline{F}_a \neq \underline{G}_b) > \underline{P}(\underline{F}_a \neq \underline{G}_{b'}) + \underline{P}(\underline{F}_{a'} \neq \underline{G}_b) + \underline{P}(\underline{F}_{a'} \neq \underline{G}_{b'})$$

with $\underline{P}(\underline{F}_a = f, \underline{G}_b = g) = \sin^2(a-b)$ Q.E.D.



$$\underline{P}(\underline{F}_a \neq \underline{G}_b) \overset{?}{\leq} \underline{P}(\underline{F}_a \neq \underline{G}_{b'}) + \underline{P}(\underline{F}_{a'} \neq \underline{G}_b) + \underline{P}(\underline{F}_{a'} \neq \underline{G}_{b'})$$

for $a = 0$, $b = x$, $a' = b' = 3x$ and $\underline{P}(\underline{F}_a = f, \underline{G}_b = g) = \sin^2(a-b)$

this inequality reads $f(x) \overset{?}{\geq} 0$, which is not the case for all x!

EPR-BOHM FOR MASSIVE SPIN-ONE PARTICLES

EPR-Bohm experiment has been done with photons (Aspect)
Setting were just *angles* a (for Alice) and b (for Bob)

- Free Will Theorem assumes it works for massive spin-1 particles:

  Settings now *orthonormal bases* $\mathbf{e} = (e_1, e_2, e_3)$ in $\mathbb{R}^3$: $A = \mathbf{a}$, $B = \mathbf{b}$

  Alice (Bob) simultaneously measures (commuting) squares of components $\langle J, a_i \rangle^2$, $i = 1, 2, 3$, of angular momentum J along $\mathbf{a}$ ($\mathbf{b}$)

- QM (theory of massive spin-**1**): possible values of $F = (F_1, F_2, F_3)$ and $G = (G_1, G_2, G_3)$ must be (1,1,0) or (1,0,1) or (0,1,1) ($\times \hbar$)

- QM (perfect EPR-correlations for a = b): if $a_i = b_j$, then $F_i = G_j$

CONWAY-KOCHEN ‘FREE WILL THEOREM’

The following assumptions are contradictory:

- *Determinism*: there exist a state space Ω and functions (F, G, A, B) such that $x \in \Omega$ determines the settings $\mathbf{a} = A(x)$, $\mathbf{b} = B(x)$, and the outcomes F(x), G(x) of the experiment

- *Free Choice*: there exists a function Z on Ω such that $F = F(A, B, Z)$, $G = G(A, B, Z)$, and for any joint value $(\mathbf{a}, \mathbf{b}, z)$ there exists $x \in \Omega$ such that $\mathbf{a} = A(x)$, $\mathbf{b} = B(x)$, $z = Z(x)$

- *Locality*: $F(A, B, Z) = F(A, Z)$ and $G(A, B, Z) = G(B, Z)$

- *QM of EPR-correlated massive Spin-***1** *particles:* as above

PROOF OF THE 'FREE WILL THEOREM'

- *Determinism* + *Free Choice* imply that $F = \underline{F}(\mathbf{a}, \mathbf{b}, z)$ and $G = \underline{G}(\mathbf{a}, \mathbf{b}, z)$, where $(\mathbf{a}, \mathbf{b}, z)$ are *free* variables
- *Locality* implies $\underline{F}(\mathbf{a}, \mathbf{b}, z) = \underline{F}(\mathbf{a}, z)$ and $\underline{G}(\mathbf{a}, \mathbf{b}, z) = \underline{G}(\mathbf{b}, z)$
- *QM*: $(a_i = b_j \Rightarrow F_i = G_j)$ for $i = j$ implies $\underline{F}_i(a_1, a_2, a_3, z) = \underline{F}_i(a_i, z)$, and for $i \neq j$ implies $\underline{F}_1(a, z) = \underline{F}_2(a, z) = \underline{F}_3(a, z) \equiv F_z(a) = F_z(-a)$
- *QM*: $F = (1,1,0)$ or $(1,0,1)$ or $(0,1,1) \Rightarrow$ function F_z from unit vectors in $\mathbb{R}^3$ to $\{0,1\}$ assigns two 1's and one 0 to each o.n.b.
- *Kochen-Specker Theorem* (1967): such functions do not exist

Simon Kochen & John Conway

Waterfall
by M.C. Escher

Ernst Specker & Simon Kochen

CONCLUSION

Both theorems prove contradiction between:

Determinism - Free Choice - Locality - Nature

Bell'Theorem shows incompatibility with Nature as represented by *probabilistic outcomes* of EPR-experiment with photons

Free Will Theorem shows incompatibility with Nature given by *individual outcomes* of EPR-experiment with spin-1particles

Which assumption to reject? All possibilities occur!

- Most physicists (and Conway & Kochen!) reject Determinism
- Nobel Laureate Gerard ‘t Hooft rejects Free Choice
- Followers of David Bohm (like John Bell) reject Locality

A PHILOSOPHICAL ERROR?

Conway & Kochen paraphrase their Free Will Theorem as :

"If we humans have free will, then elementary particles already have their own small share of this valuable commodity"

(Free Choice + Locality + Nature $\Rightarrow \neg$ Determinism)

- Free Choice mistaken for Free Will
- Free Will supposed to make sense without Determinism
- Free Will Theorem defines Free Choice *given* Determinism

ORIGINS OF THE FREE WILL THEOREM

Suppose *free* setting of experiment is an n x n *matrix a*

- Von Neumann (1932) proved impossibility of a (free) deterministic theory for which outcome $\underline{F}(a, z)$ is linear in *a*
- This was criticized since even eigenvalues are not linear in *a*
- Kochen & Specker (1967) proved this under the significantly weaker assumption that $\underline{F}(a, z)$ is linear in *commuting* matrices
- This was criticized since $\underline{F}$ should be allowed to also depend on 'measurement context' i.e. on matrices commuting with *a*
- Conway & Kochen (2009) removed also the latter objection (at the price of introducing a locality condition)