On the role of time in Bell's theorem, & some thoughts on Bell-denialism, Part 1

QM foundations - nature of time seminar, J U Krakow

Richard Gill

This, part 1: Monday 6 April 2020, 17:00 Warsaw time, Zoom

Part 2: **Tuesday** 14 April 2020, 17:00 Warsaw time, Zoom

This file: https://www.math.leidenuniv.nl/~gill/Warsaw.pdf

Part 2: https://www.math.leidenuniv.nl/~gill/Warsaw2.pdf

In memoriam Boris Tsirelson (1950 – 2020)

My scientific career (part 1)

Mathematical Statistician, b. 1951, England

- Studied maths at Cambridge
- Dutch girlfriend
- PhD in Amsterdam (CWI/VU, 1979)
- Head of dept of Math Statistics; visiting professor in Leiden 1985
- Utrecht … Leiden … now emeritus prof.

- Asymptotic statistics esp. semiparametric models, survival analysis
- Martingale methods in statistics

My scientific career (part 2)

Mathematical Statistician, b. 1951, England

- Statistical issues in miscarriages of justice: serial killer nurses, forensic DNA
- Scientific integrity (esp. fake statistics; dubious research practices)
- Statistical issues in matters of public debate

- Quantum tomography (Q. State reconstruction) as a statistical problem (error bars, hypothesis tests; infinite dimensional parameter)
- Quantum foundations esp. Bell's theorem, Bell inequalities

- Mathematical diversions (three doors, two envelopes, ...)
- Statistical consultation, mushroom foraging, Buddhism, psychology, ...

How I got into Q?

Bell's theorem!

- Luigi Accardi, Hans Maassen: there is probability in QM, but it's *a different kind of probability*
- My reaction: **this is nonsense**. (Frequentist) Kolmogorov probability is formalisation of elementary *book-keeping* rules
- There is no *mathematical* reason whatsoever why probability models of different (repeatable) experiments should be parts of of one bigger model; there can only be *physical* reasons
- Bell doesn't tell us that we need different *mathematical rules*; he shows there is an issue with certain *physical* assumptions

"ω" is just a label, not a physical variable!

Engagement with Bell deniers (1)

Issues of time & statistics: Luigi Accardi; Hess& Philipp; …

- Gill, Weihs, Zeilinger & Żukowski (2002, 2003)
- Larsson & Gill (2004)
    - New loopholes (the "coincidence loophole")
- Acín, Gill, Gisin (2005)
    - CGLMP inequality demystified
- Computer challenges ("quantum Randi challenges")
- Martingale tests in Bell-CHSH type experiments (protect against memory loophole, time trends and time shifts, …)
    - Used in all 2015 "loophole free Bell experiments" (Vienna, NIST, Delft, Munich)
- In preparation: optimising statistical analysis of those experiments (smaller *p*-values)

Engagement with Bell deniers (2)

The detection loophole; Joy Christian; …

- Gill (2020a, b, c)
  - Pearle's model, Steve Gull's proof, Joy Christian's chicanery

Scientific director

Einstein Centre for Local Realistic Physics

University of Oxford [?]

Bell's inequality, Bell's theorem

From EPR to Bell-CHSH and beyond

- EPR → EPR-B → Bell's 3 correlations inequality → Bell-CHSH (4 correlations) inequality and Bell's theorem
- Bell's theorem:
  - Executive summary:
    QM is incompatible with [locality ∩ realism ∩ freedom]
  - A loophole-free experiment can show that physical reality is incompatible with [locality ∩ realism ∩ freedom]
  - NB this is a mathematical theorem about mathematical models: 'physical interpretation' / metaphysics / philosophy is for physicists and/or philosophers of science to discuss

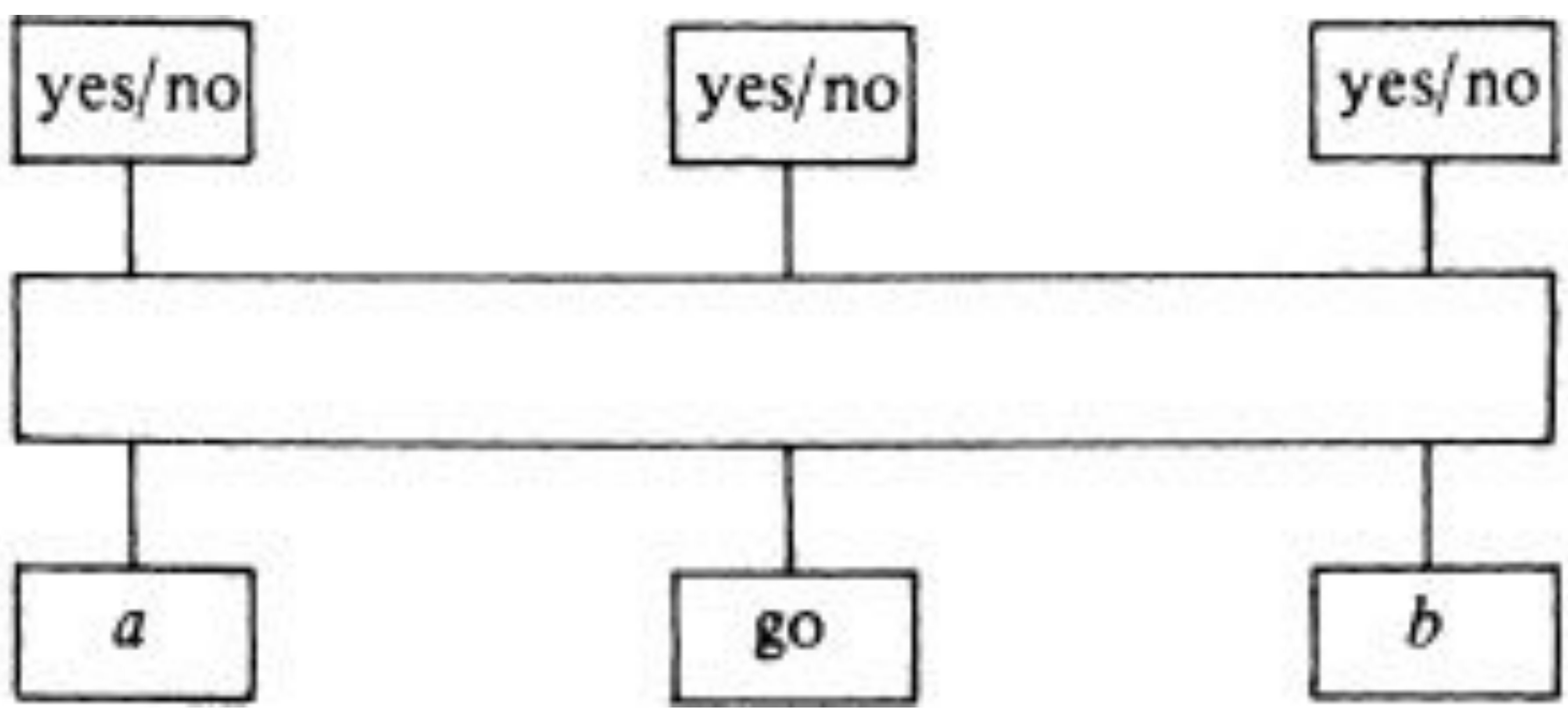
Loophole free experiments

Bertlmann's socks

2 binary outputs - outcomes $\in \{-1, +1\}$

$X = x$

$Y = y$



$A = a$

2 binary inputs - settings $\in \{1, 2\}$

$B = b$

Time

Bertlmann's socks (1)

"Finally you might suspect that the very notion of particle, and particle orbit, freely used above in introducing the problem, has somehow led us astray. Indeed did not Einstein think that fields rather than particles are at the bottom of everything? So the following argument will not mention particles, nor indeed fields, nor any other particular picture of what goes on at the microscopic level. Nor will it involve any use of the words 'quantum mechanical system', which can have an unfortunate effect on the discussion. The difficulty is not created by any such picture or any such terminology. It is created by the predictions about the correlations in the visible outputs of certain conceivable experimental set-ups."

Bell (1981) https://hal.archives-ouvertes.fr/jpa-00220688/document

Bertlmann's socks (2)

"Consider the general experimental set-up of Fig. 7. To avoid inessential details it is represented just as a long box of unspecified equipment, with three inputs and three outputs. The outputs, above in the figure, can be three pieces of paper, each with either a 'yes' or a 'no' printed on it. The central input is just a 'go' signal which sets the experiment off at time t_1. Shortly after that the central output says 'yes' or 'no'. We are only interested in the 'yes's, which confirm that everything has got off to a good start (e.g., there are no 'particles' going in the wrong directions, and so on). At time $t_1 + T$ the other outputs appear, each with 'yes' or a 'no' (depending for example on whether or not a signal has appeared on the 'up' side of a detecting screen behind a local Stern-Gerlach magnet). The apparatus then rests and recovers internally in preparation for a subsequent repetition of the experiment. But just before time $t_1 + T$, say at time $t_1 + T - \delta$, signals a and b are injected at the two ends. (They might for example dictate that Stern-Gerlach magnets be rotated by angles α and β away from some standard positions). We can arrange that $c\delta \ll L$, where c is the velocity of light and L the length of the box; we would not then expect the signal at one end to have any influence on the output at the other, for lack of time, whatever hidden connections there might be between the two ends."

$$c = 3\times10^9\ m\ s^{-1} \qquad L = 1500\ m \qquad \delta \ll 0.5\times10^{-6}\ s$$

Bell's inequality, Bell's theorem

- Settings

  A, B take values in $\{1, 2\}$

- Counterfactual outcomes

  $X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{21}, Y_{22}$ take values in $\{-1, +1\}$

- Actual outcomes

  $X = X_{AB}, Y = Y_{AB}$

$$X(\omega) = X_{A(\omega)B(\omega)}(\omega)$$

- **Freedom (no conspiracy)**

  $(A, B) \perp\!\!\!\perp (X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{21}, Y_{22})$

$\perp\!\!\!\perp$ means 'statistically independent'

Locality, realism, freedom

- **Realism** := "existence" of counterfactual outcomes

- **Locality** := Alice's outcomes don't depend on Bob's settings & vice-versa

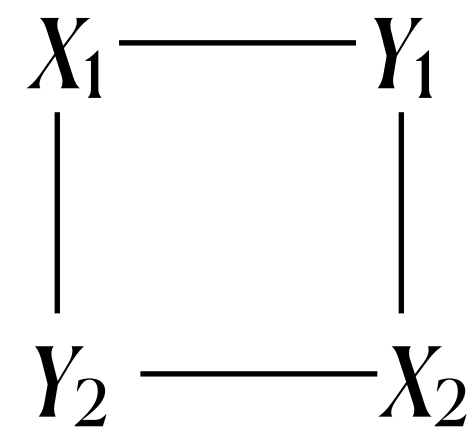
$$X_{11} = X_{12} =: X_1, \quad X_{21} = X_{22} =: X_2, \quad X := X_A$$

$$Y_{11} = Y_{21} =: Y_1, \quad Y_{12} = Y_{22} =: Y_2, \quad Y := Y_B$$

- **Freedom** := statistical independence of **actual** settings from **counterfactual** outcomes

$$(A, B) \perp\!\!\!\perp (X_1, X_2, Y_1, Y_2)$$

Bell's inequality, Bell's theorem



$$X_1 = Y_2 \;\;\&\;\; Y_2 = X_2 \;\;\&\;\; X_2 = Y_1 \;\;\Rightarrow\;\; X_1 = Y_1$$

$$\therefore\;\; X_1 \neq Y_1 \;\;\Rightarrow\;\; X_1 \neq Y_2 \;\;\text{or}\;\; Y_2 \neq X_2 \;\;\text{or}\;\; X_2 \neq Y_1$$

$$\therefore\;\; \mathrm{P}(X_1 \neq Y_1) \;\leq\; \mathrm{P}(X_1 \neq Y_2) \;+\; \mathrm{P}(Y_2 \neq X_2) \;+\; \mathrm{P}(X_2 \neq Y_1)$$

$$\therefore\;\; \mathrm{P}_{11}(X \neq Y) \;\leq\; \mathrm{P}_{12}(X \neq Y) \;+\; \mathrm{P}_{22}(Y \neq X) \;+\; \mathrm{P}_{21}(X \neq Y)$$

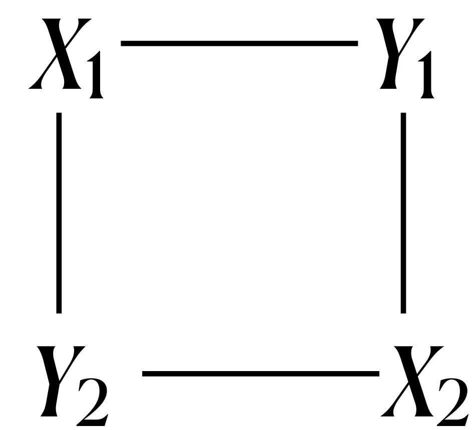
where

$$\mathrm{P}_{ab}(\ldots) := \mathrm{P}(\;\ldots \mid A = a, B = b)$$

by freedom!

Bell's inequality is a trivial probabilistic corollary of a trivial <u>logical implication !</u>

Bell's inequality, Bell's theorem



$$P_{11}(X \neq Y) \;\leq\; P_{12}(X \neq Y) + P_{22}(Y \neq X) + P_{21}(X \neq Y)$$

For instance: $0.75 \leq 0.25 + 0.25 + 0.25$

But QM promises we can get (approx.) 0.85, 0.15, 0.15, 0.15

Notice: $E_{ab}(XY) = P_{ab}(X = Y) - P_{ab}(X \neq Y) = 1 - 2\,P_{ab}(X \neq Y)$

Define: $S := E_{12}(XY) + E_{22}(XY) + E_{21}(XY) - E_{11}(XY) =: \rho_{12} + \rho_{22} + \rho_{21} - \rho_{11}$

Then, equivalently: $S \leq 2$ (CHSH: Clauser-Horne-Shimony-Holt, 1969)

QM promises we can (maximally) get $2\sqrt{2}$ (why not 4 ?)

Tsirelson, 1980

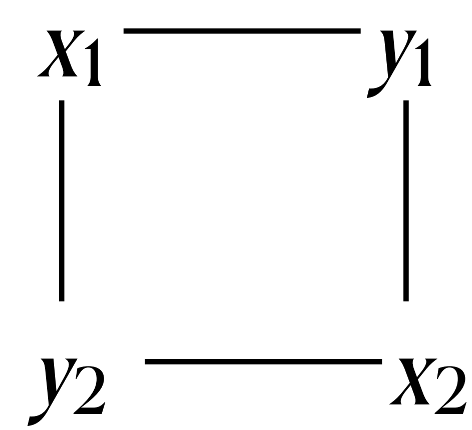
Bell, 1964

Pawlowski et al., 2009

Turning around the randomness:

Randomise **the settings,** ***fix*** **the counterfactual outcomes!**

- Suppose A, B are independent, fair, Bernoulli



- Let $I_{ab} := I(A = a, B = b)$, $\mathrm{E}(I_{ab}) = 0.25$

- **Condition** on past & on $(X_1, X_2, Y_1, Y_2) = (x_1, x_2, y_1, y_2)$,
  define: $\delta_{ab} := I(x_a \neq y_b)$

- Observe that

$$\mathrm{E}(\delta_{11} I_{11} - \delta_{12} I_{12} - \delta_{22} I_{22} - \delta_{21} I_{21} \mid \dots) \leq 0$$

because

$$x_1 \neq y_1 \Rightarrow x_1 \neq y_2 \text{ or } y_2 \neq x_2 \text{ or } x_2 \neq y_1$$

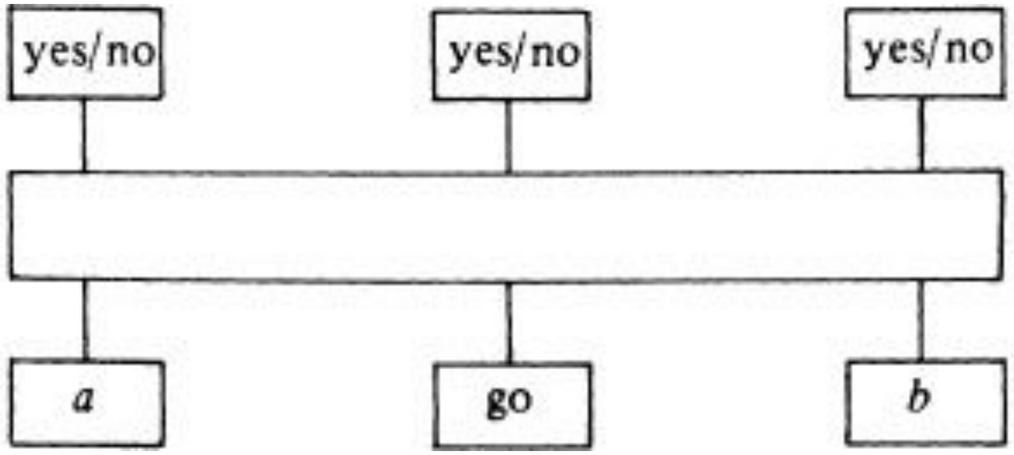
<u>This leads to martingale tests</u>: with protection against: time dependence, time trends and jumps, opportunistic stopping or skipping ...

Bell games

- The experiment consists of a long *run* of N *trials*
- **Each** trial, *settings* $A, B \in \{1, 2\}$ chosen **anew** by fair coin tosses; observe *outcomes* X, Y
- A trial results in a *success* if and only if:

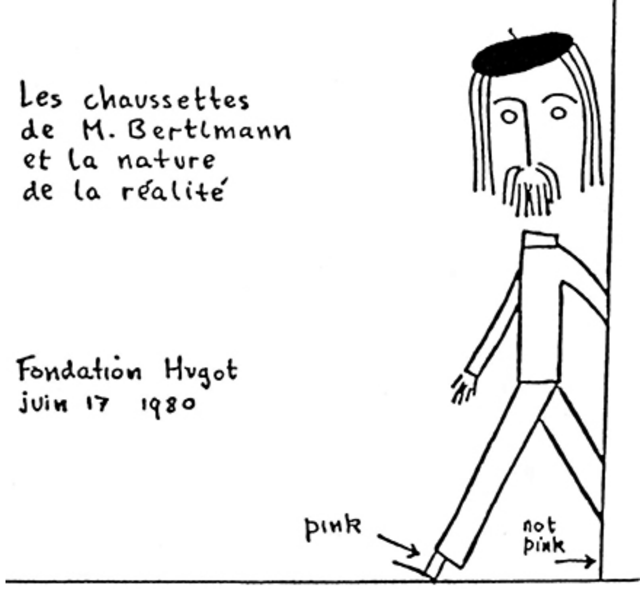
  not[$A = B = 1$] **&** [$X = Y$], **or** [$A = B = 1$] **&** **not**[$X = Y$]
- **Theorem**: under local realism (for each trial, conditional on past)

  $\forall\, x :$ Prob(#Successes $\geq x$) $\leq$ Prob(Binom(N, 0.75) $\geq x$))
- Under QM, we can attain Binom(N, 0.85)



Bell games

Who is playing against whom?



- The Bell-denialist claims to have a local realistic model which reproduces the single correlations, or violates the CHSH inequality
- They must program their model
- I insist on supplying the settings
- I will test that their programs satisfy the requirements needed so that it genuinely models an experiment of the type specified by Bell

References

… and discussion

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Statistics, Causality and Bell's Theorem

Richard D. Gill

Bell's [*Physics* **1** (1964) 195–200] theorem is popularly supposed to establish the nonlocality of quantum physics. Violation of Bell's inequality in experiments such as that of Aspect, Dalibard and Roger [*Phys. Rev. Lett.* **49** (1982) 1804–1807] provides empirical proof of nonlocality in the real world. This paper reviews recent work on Bell's theorem, linking it to issues in causality as understood by statisticians. The paper starts with a proof of a strong, finite sample, version of Bell's inequality and thereby also of Bell's theorem, which states that quantum theory is incompatible with the conjunction of three formerly uncontroversial physical principles, here referred to as locality, realism and freedom. Locality is the principle that the direction of causality matches the direction of time, and that causal influences need time to propagate spatially. Realism and freedom are directly connected to statistical thinking on causality: they relate to counterfactual reasoning, and to randomisation, respectively. Experimental loopholes in state-of-the-art Bell type experiments are related to statistical issues of post-selection in observational studies, and the missing at random assumption. They can be avoided by properly matching the statistical analysis to the actual experimental design, instead of by making untestable assumptions of independence between observed and unobserved variables. Methodological and statistical issues in the design of quantum Randi challenges (QRC) are discussed. The paper argues that Bell's theorem (and its experimental confirmation) should lead us to relinquish not locality, but realism.

Statistical Science 2014, **29** (4) 512–528

DOI: 10.1214/14-STS490

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