Maximal Passion at a Distance

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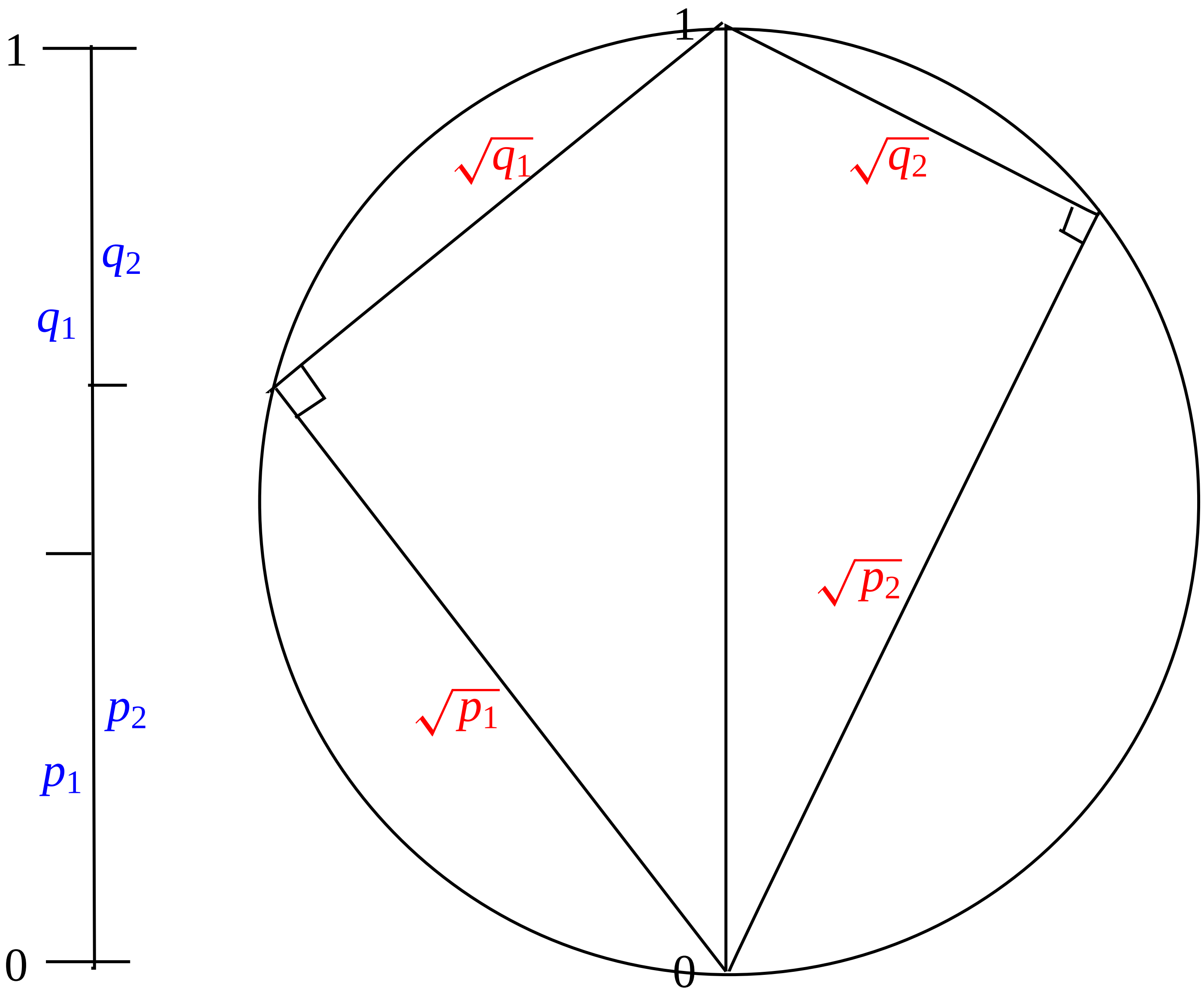
ETH Zürich, 12 January 2007

Bell's 1964 inequality $\rightarrow$ {
- mathematical proof of *Bell's theorem*
- CHSH inequality, 1969
- experimental proof of *quantum nonlocality**, Aspect et al 1982, ...

* "quantum nonlocality" means: "quantum induced apparent classical nonlocality"

Nature's two ways



$p_1 + q_1 = 1 = p_2 + q_2$

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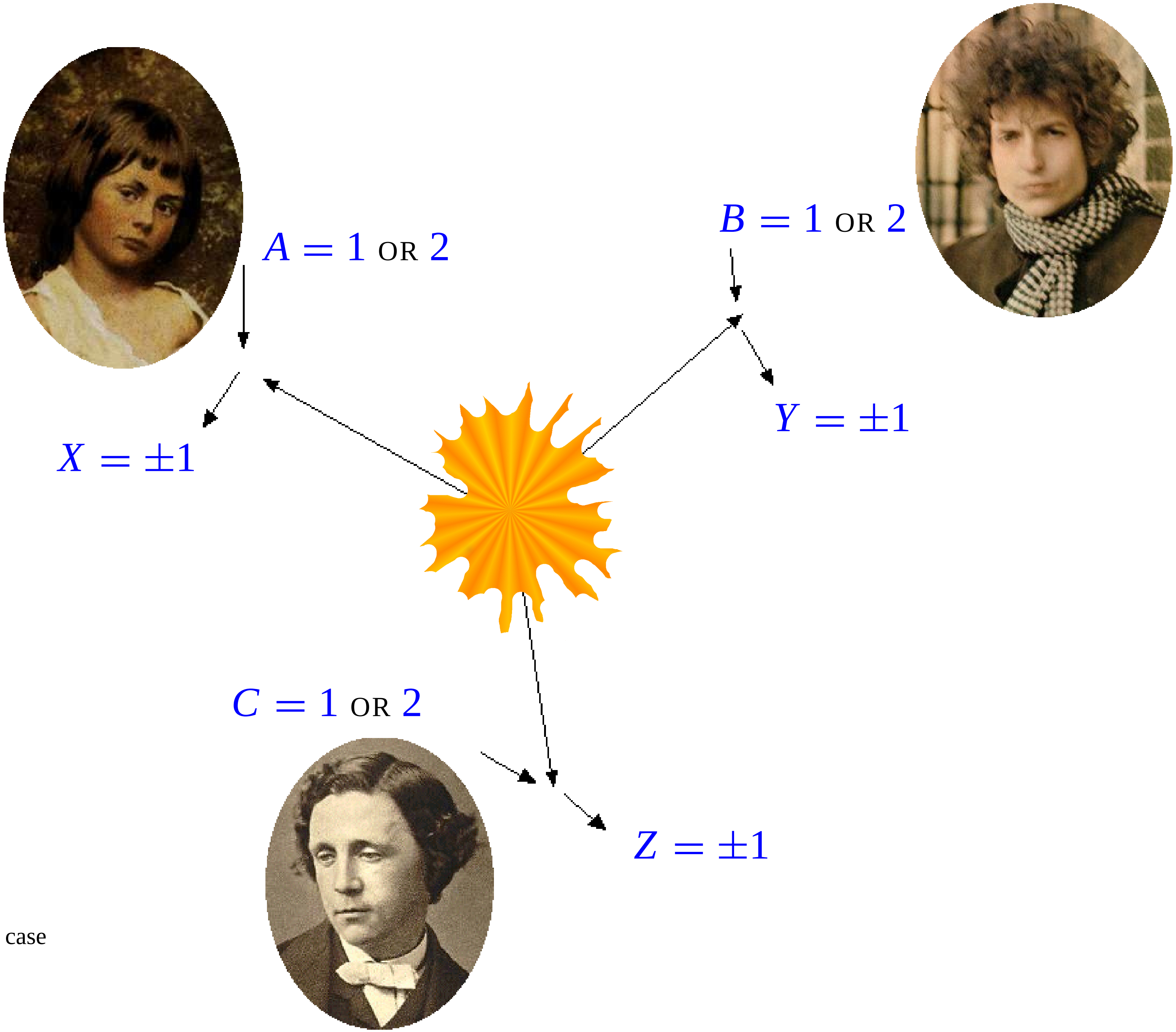
GHZ 1988 paradox, Pan et al. 2000 experiment

3 parties × 2 settings × 2 outcomes



CHSH, Aspect: 2 × 2 × 2 case

Overview

* Better Bell experiments! (Bell *and* Cirelson)

* classical polytope $\subset$ quantum convex body $\subset$ no-signalling polytope

* which criterion (Euclidean distance $\rightarrow$ relative entropy)

* `arXiv.org:math.ST/0610115`, `http://www.math.leidenuniv.nl/~gill`
Passion at a Distance: Better Bell Inequalities

Case studies

1) orig Bell vs CHSH vs Hardy vs GHZ, *Bell without inequalities*

2) qutrits and beyond: CGLMP

3) Hardy ladder proofs

4) new CH-like inequality for binary outcomes plus undetected non-detections

Notation

Fix # parties, # settings, # outcomes

* $p\,(x\ y\ ..\ |\ a\ b\ ..)$, $q\,(x\ y\ ..\ |\ a\ b\ ..)$

probability of joint outcomes $x\ y\ ..$ given joint settings $a\ b\ ..$

under classical, resp. quantum theory

* $\pi\ (a\ b\ ..)$ probability of settings $a\ b\ ..$; chosen by experimenter;

mostly: kept fixed

* $p\,(a\ b\ ..\ x\ y..) = \pi\ (a\ b\ ..)\ p\,(x\ y\ ..\ |\ a\ b\ ..)$ and
$q\,(a\ b\ ..\ x\ y..) = \pi\ (a\ b\ ..)\ q\,(x\ y\ ..\ |\ a\ b\ ..)$

defines vectors $\vec{p}$ and $\vec{q}$; sets $\{\vec{p}\}$ and $\{\vec{q}\}$

Key facts

No-signalling affine subspace $\supset$ no-signalling polytope

$\supset$ quantum convex body $\{\vec{q}\}$ $\supset$ classical polytope $\{\vec{p}\}$

$\ni$ completely random point

Best experiment $\vec{q}$ for given # parties, # settings, # outcomes, solves

$$\sup_{q} \inf_{p} \sum_{a\,b\,..\,x\,y\,..} q(a\,b\,..\;x\;y\,..)\;\log_2 \frac{q(a\,b\,..\;x\;y\,..)}{p(a\,b\,..\;x\;y\,..)}$$

more precisely:

sup over experimental parameters: state, measurements, joint setting probabilities

inf over classical theories
computed by missing-data maximum-likelihood (Groeneboom; programs)

Changing the range of the optimization in various ways leads to
non-locality measures for states, set-ups, . . .

The classical polytope $\{\vec{p}\}$

aka “local realism”, “the local polytope”, “local hidden variables”

$$\exists \ (X_a \, Y_b \, ..)_{ab..} \quad \text{such that} \quad \forall_{ab..} \ \big(p\,(xy.. \mid ab..)\big)_{xy..} = \text{law}(X_a \, Y_b ..)$$

Results of unperformed experiments exist, too

Counterfactual outcomes of nonmeasured observables

The quantum body $\{\vec{q}\}$

$\exists$ closed subspaces $L_x^a\ M_y^b\ ..$ of Hilbert spaces $\mathscr{H}\ \mathscr{K}\ ..$ such that

* $\forall_a \quad (L_x^a)_x$ is an orthogonal decomposition of $\mathscr{H}$
* $\forall_b \quad (M_y^b)_y$ is an orthogonal decomposition of $\mathscr{K}$
* $..$

$\exists\ \Psi \in \mathscr{H} \otimes \mathscr{K} \otimes .. \quad \|\Psi\|^2 = 1$ such that

* $q\ (xy..\ |\ ab..) \ = \ \|\Pi_{L_x^a \otimes M_y^b \otimes ..} \Psi\|^2$

The quantum body $\{\vec{q}\}$

$\exists$ observables (self-adjoint operators) X_a Y_b ..
such that each X_a commutes with each Y_b, each .. , and
$\exists$ a state Ψ such that

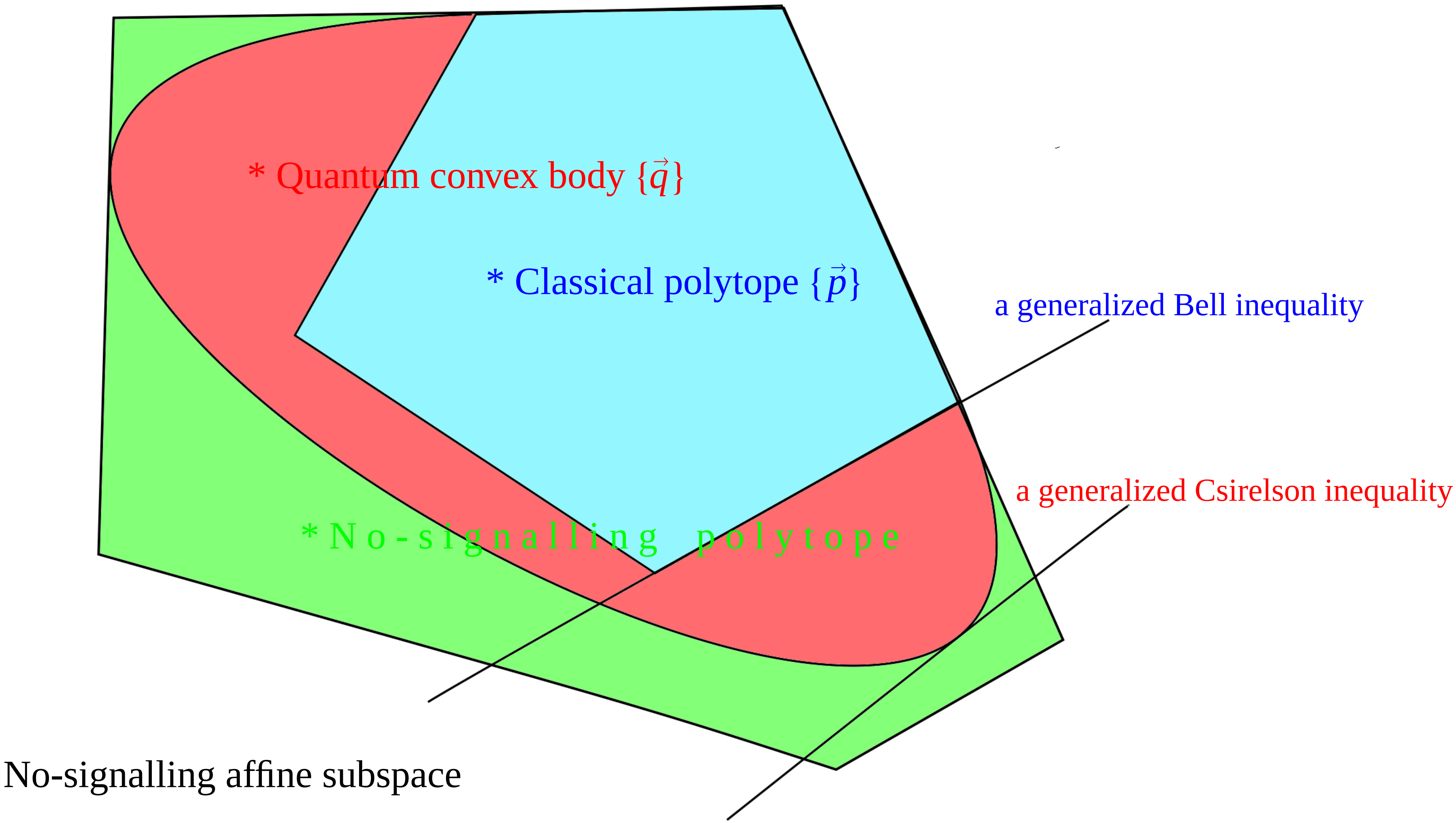
$$* \quad q\,(xy..\,|\,ab..) \;=\; \|\Pi_{\{X_a=x\}\cap\{Y_b=y\}\cap\,..}\;\Psi\|^2$$

The classical polytope $\{\vec{p}\}$

All X_a Y_b .. commute

OR

There exist random variables and a probability measure . . .


* Quantum convex body $\{\vec{q}\}$
* Classical polytope $\{\vec{p}\}$
a generalized Bell inequality
a generalized Csirelson inequality
* N o - s i g n a l l i n g   p o l y t o p e
No-signalling affine subspace

GHZ paradox, Pan et al. experiment

Suppose $X_a^2 \equiv Y_b^2 \equiv Z_c^2 \equiv 1$

Classical :

If $Y_2 Y_1 = Y_1 Y_2$ then $(X_1 Y_2 Z_2)(X_2 Y_1 Z_2)(X_2 Y_2 Z_1) = (X_1 Y_1 Z_1)$

So $X_1 Y_2 Z_2 \equiv X_2 Y_1 Z_2 \equiv X_2 Y_2 Z_1 \equiv +1 \implies X_1 Y_1 Z_1 \equiv +1$

Quantum :

But if $Y_2 Y_1 = -Y_1 Y_2$ then $(X_1 Y_2 Z_2)(X_2 Y_1 Z_2)(X_2 Y_2 Z_1) = -(X_1 Y_1 Z_1)$

So $X_1 Y_2 Z_2 \equiv X_2 Y_1 Z_2 \equiv X_2 Y_2 Z_1 \equiv +1 \implies X_1 Y_1 Z_1 \equiv -1$

This can be arranged
- theoretically: GHZ
- experimentally: Pan, Bouwmeester, ...

Case study 1: *Comparison of famous experiments*

Peres 2000; van Dam, Gill, Grünwald 2005; Groeneboom et al

eg: GHZ is potentially 9.0 times better than CHSH
but as now implemented only 9/8 times better per singlet

even assuming perfect state generation and measurement

Case study 2: $2 \times 2 \times d$ *qutrits and beyond*

CGLMP 2002, Acin, Gill & Gisin 2006, Gill & Zohren

Compare original form and derivation CGLMP (2002) *Phys Rev Lett* `quant-ph/0106024` with

$$X_1 \leq Y_2 \text{ and } Y_2 \leq X_2 \text{ and } X_2 \leq Y_1 \quad \text{implies} \quad X_1 \leq Y_1$$

<u>Therefore</u> $\quad X_1 > Y_1 \quad \text{implies} \quad X_1 > Y_2 \text{ or } Y_2 > X_2 \text{ or } X_2 > Y_1$

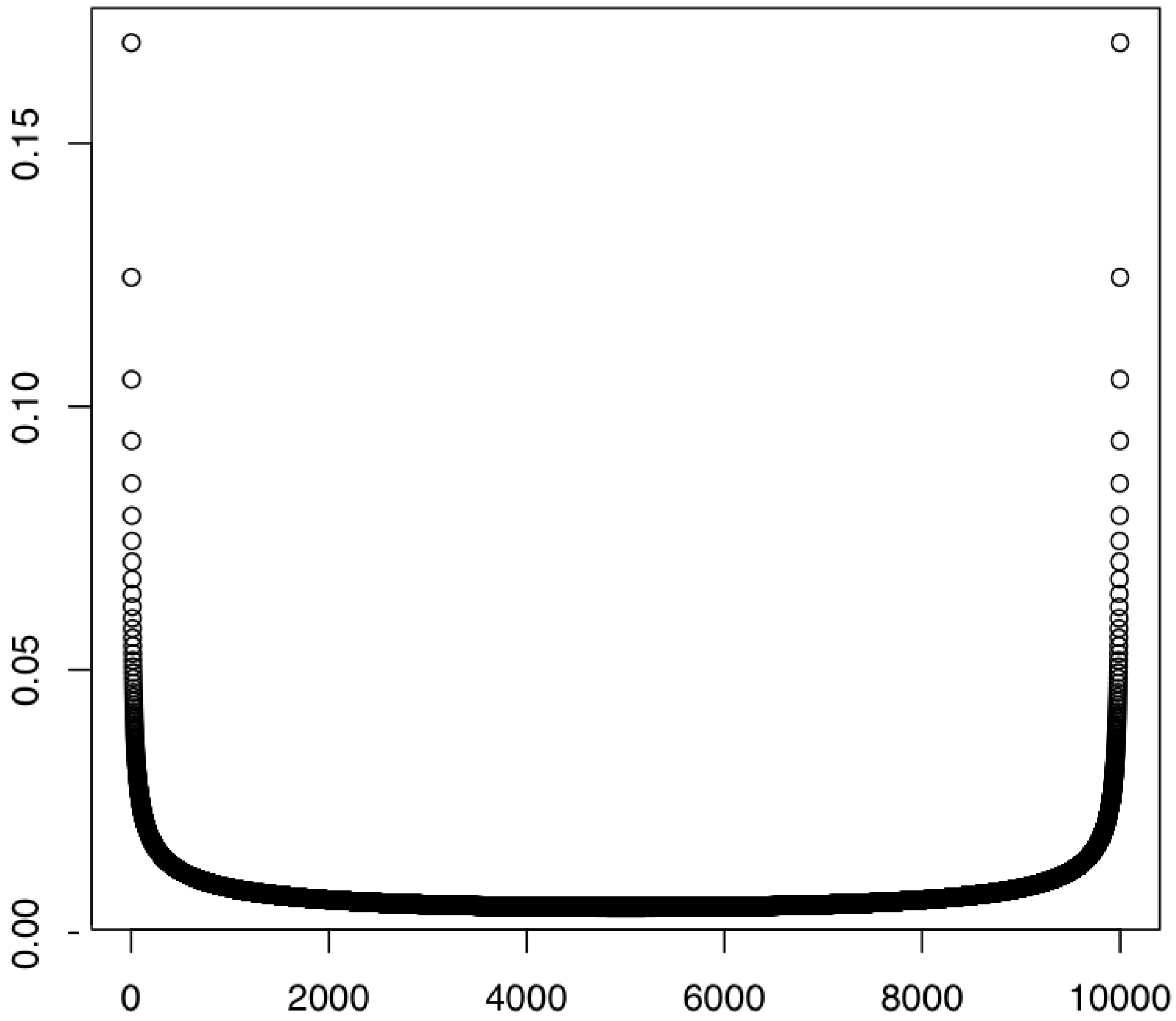
<u>Therefore</u> $\quad \Pr(X_1 > Y_1) \leq \Pr(X_1 > Y_2) + \Pr(Y_2 > X_2) + \Pr(X_2 > Y_1)$

Numerics:

best *state* is *not the maximally entangled state*

best measurements are “QFT with CHSH phases” (generalized CHSH)

Using *Ansatz* and tricks from convexity and optimization
can compute up to $d = 10\,000$ (see plot, next slide)



$d = 10\,000$: Schmidt coefficients of best state (wrt relative entropy)

Conjectured best CGLMP state, measurements

$$|\Psi\rangle = \sum_{j=0}^{d-1} c_j \, |j\ j\rangle$$ for certain $c_j \geq 0$, U-shaped

Alice chooses $\alpha = 0$ or $\pi/2$

Bob chooses $\beta = \pi/4$ or $-\pi/4$

Alice, Bob apply diagonal unitaries $e^{i\frac{j\alpha}{d}}$, $e^{i\frac{j\beta}{d}}$

Alice does QFT, Bob $\mathrm{QFT}^\dagger$, they measure in computational basis

Are these state and (generalized CHSH) measurements,
the best state and measurements, for CGLMP?

Proof?

$d = 2$
Tsirel'son; Euclidean distance only

$d = 3$
Navascues Pironio Acin `quant-ph/0607119`; Euclidean distance only

new upper bound, numerically attained
ie, proof to within precision ϵ

$d > 3$
only numerical evidence, increasingly sparse and/or
depending on unproven-assumption justified numerics

I believe that Stefan Zohren's new form of CGLMP might help us
prove the optimality of the generalized CHSH measurements and obtain properties
of the optimal state ...

CGLMP conjectures

Conjecture 1:

in 2 party $\times$ 2 setting $\times$ d outcome polytope, all *interesting* faces are CGLMP faces

Conjecture 2:

* optimal state is not maximally entangled

* optimal measurements are generalized CHSH

Conjecture 3:

For $d \to \infty$ can achieve $1 \leq 0 + 0 + 0$ in

$$\Pr(X_1 > Y_1) \leq \Pr(X_1 > Y_2) + \Pr(Y_2 > X_2) + \Pr(X_2 > Y_1)$$

Polish/Russian poker: you choose Y_1 or Y_2; I choose X_1 or X_2; I always beat you

Case study 3: new *Hardy Ladder proofs*

With Marco Barbieri

Add to CGLMP inequality, same with settings 1, 2 replaced by 2, 3
and again . . .

Cancellation gives a kind of ladder or cat's cradle
crossing pairs of adjacent measurement settings remain
intermediate horizontal rungs are deleted

eg, best ladder has length 5 and uses 10 of the $5 \times 5 = 25$ setting pairs

Best correlated settings:
much better experiment than CHSH

Best independent settings, so 15 superfluous pairs:
worse experiment than CHSH

Must adjust vDGG's conjecture (# 3) :
optimal experiment with Bell singlet state is CHSH . . .

Case study 4: *new CH-like CHSH inequality for detector inefficiency*

With Jan-Åke Larsson

$2 \times 2 \times 3$ case — third outcome is no-detection

Define $\vec{q}$ by adding probability ϵ of no-detection to CHSH

Relative-entropy-closest $\vec{p}$ lies on face of classical polytope
a generalized Bell inequality

Equality constraints (affine subspace) $\longrightarrow$
inequality in "observable" probabilities *only*
cf. CHSH $\longrightarrow$ CH inequality

Related to Larsson's *nonlinear* detector-inefficiency adjusted Bell inequality
but linear

Martingale methods demonstrate insurance against memory loophole
Gill 2003, Växjö

Notes

Recall the quantum information open problems site of Reinhard Werner

Problems # 26 and # 27 are mine
see also # 1 generate all Bell inequalities
but don't give this *problem to your PhD student*

Government Health Warning:

Anyone who engages in the interpretation of quantum mechanics disappears into a black hole and is never heard of again
(R. Feynmann)

The next slides could cause serious damage to your mind

Will a succesful loophole free experiment ever be performed?

A few sensible people think *no*, eg, Emilio Santos

It is *logically possible* that quantum physics itself prevents setting up "initial conditions" necessary for decisive experiment (cf. uncertainty relation)

John Bell agreed logically possible but unlikely, would be interested if experimental/theoretical support were forthcoming

This "Bell's fifth position" provides a niche for Gerard 't Hooft to craft a local hidden variables theory underlying quantum physics as we know it

My quantum philosophy

Detector clicks (macrosccopic events) in the past are real
The future is a wave of potentiality
its probability determined by quantum theory

Consciousness resides on the boundary between past and the future: now

As now moves relentlessly forward the past crystallizes out of it randomly

No nonlocality problem
No Schrödinger cat problem
No measurement problem (*time is measurement is time — creation of space-time*)

Quantum probability is ontological, not epistemological
the past is real, the wave function is real and
non-local — a function of the whole past

Quantum gravity?
There is no *quantum* gravity since space-time is classical (exists in the past only)
being randomly created now

Why is Nature like this?
The past is discrete, finite
The only way to have nature locally invariant under continuous rotations and shifts is to make it random — *probabilities* can be invariant, continuous, . . .
This leaves us with quantum theory as the only possibility

cf. Inge Helland

Inspiration:

The math: Slava Belavkin new interpretation, eventum mechanics

The imagery: Robert M. Pirsig "Zen & the art of motor cycle maintenance"

Some slogans

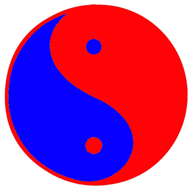
WYSIATI

Copenhagen rules: there is no *underlying* reality

The past is particles, the future is a wave
The past is discrete, the future is continuous

~~natura non fecit saltum~~
natura abhorret a vacuo

goodbye Aristoteles, Leibniz, Kant . . . ~~*everything has a cause*~~



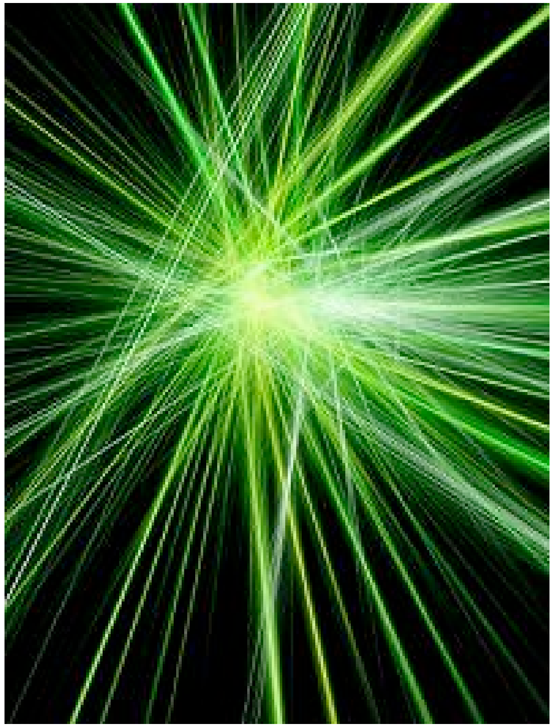
mythos versus *logos*

Yesterday is history
Tomorrow: a mystery
Today is a gift


The End

Appendix

News in Science
abc.net.au/science/news



Distance no worries for spooky particles

Stephen Pincock
ABC Science Online
Friday, 8 December 2006

A message sent using entangled, or spooky, particles of light has been beamed across the ocean *(Image: iStockphoto)*

Scientists have used quantum physics to zap an encrypted message more than 140 kilometres between two Spanish islands. Professor Anton Zeilinger from the University of Vienna and an international team of scientists used 'spooky' pulses of light to send the message. They say this is an important step towards making international communications more secure. Zeilinger described the study this week at the Australian Institute of Physics meeting in Brisbane. The photons they sent were linked together through a process known as quantum entanglement. This means that their properties remained tightly entwined or entangled, even when separated by large distances, a property Einstein called spooky. The group's achievement is important for the emerging field of quantum cryptography, which aims to use properties such as entanglement to send encrypted messages. Research groups around the world are working in this field. But until now they have only been able to send messages relatively short distances, limiting their usefulness. Zeilinger's team wants to be able to beam the messages to satellites in space, so they could theoretically be relayed anywhere on the planet.

To test their system, the team went to Tenerife in the Canary Islands, where the European Space Agency operates a telescope specifically designed to communicate with satellites. Instead of pointing the telescope at the stars, Zeilinger says, the scientists turned it to the horizontal and aimed it towards a photon sending station 144 kilometres away on the neighbouring island of La Palma. "Very broadly speaking, we were able to establish a quantum communication connection," he says. "We worried a lot about whether atmospheric turbulence would destroy the quantum states. But it turned out to work much better than we feared." The results suggest it should be possible to send encrypted photons to a satellite orbiting 300 or 400 kilometres above the Earth, he says. "This is our hope. We believe that such a system is feasible." The next step is to try the system out with an actual satellite, a project which is likely to involve the European Space Agency and others. "This is about developing quantum communications on a grand scale," Zeilinger says. His team expects to publish its results soon.

Quantum Physics, abstract
quant-ph/0607182

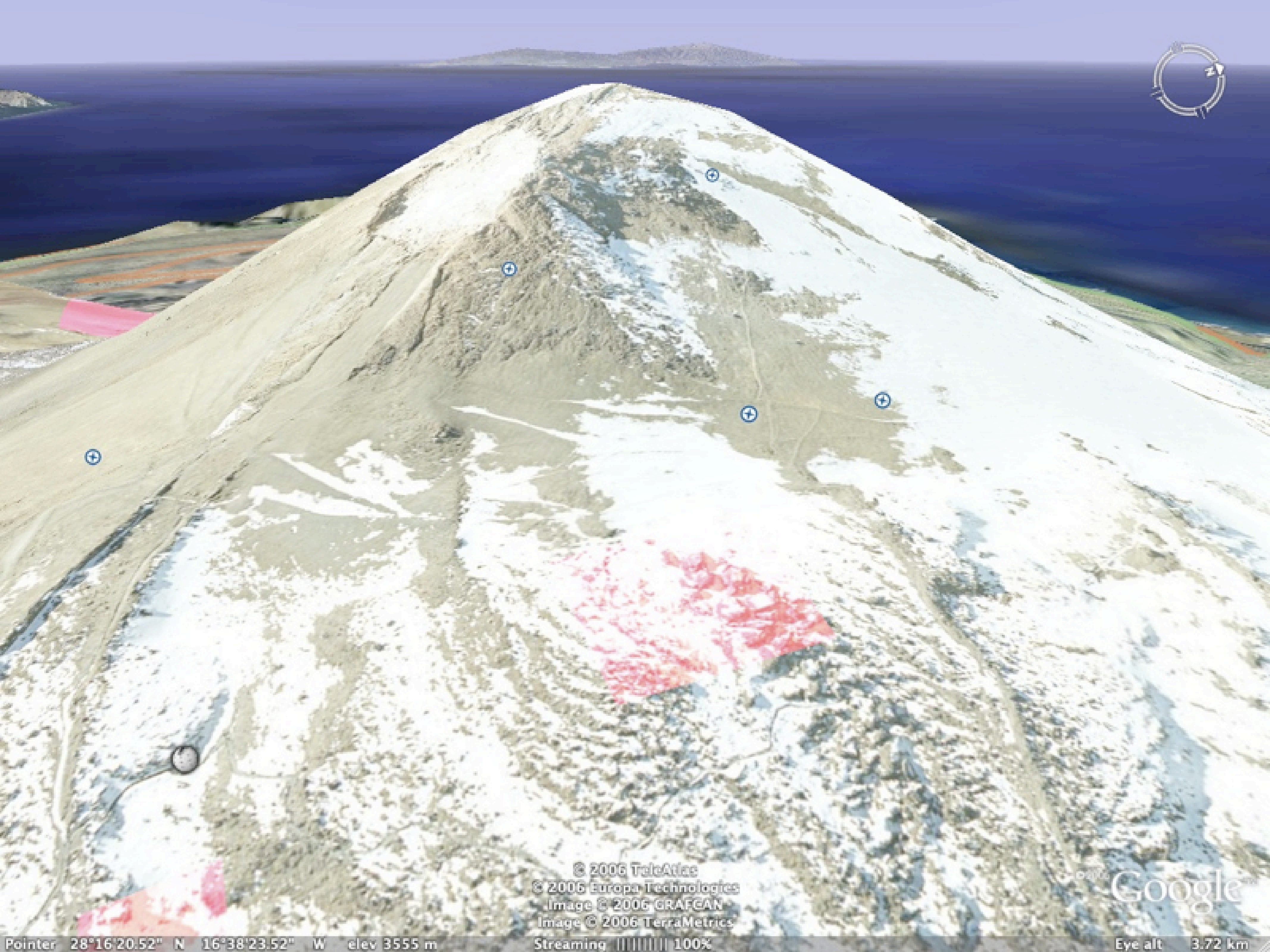
From: Rupert Ursin [view email]
Date (v1): Wed, 26 Jul 2006 14:29:14 GMT (201kb)
Date (revised v2): Thu, 27 Jul 2006 07:47:40 GMT (374kb)

Free-Space distribution of entanglement and single photons over 144 km

Authors: R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdigues, P. Trojek, B. Oemer, M. Fuerst, M. Meyenburg, J. Rarity, Z. Sodnik, C. Barbieri, H. Weinfurter, A. Zeilinger

Comments: 10 pages including 2 figures and 1 table, Corrected typos.

Quantum Entanglement is the essence of quantum physics and inspires fundamental questions about the principles of nature. Moreover it is also the basis for emerging technologies of quantum information processing such as quantum cryptography, quantum teleportation and quantum computation. Bell's discovery, that correlations measured on entangled quantum systems are at variance with a local realistic picture led to a flurry of experiments confirming the quantum predictions. However, it is still experimentally undecided whether quantum entanglement can survive global distances, as predicted by quantum theory. Here we report the violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality measured by two observers separated by 144 km between the Canary Islands of La Palma and Tenerife via an optical free-space link using the Optical Ground Station (OGS) of the European Space Agency (ESA). Furthermore we used the entangled pairs to generate a quantum cryptographic key under experimental conditions and constraints characteristic for a Space-to-ground experiment. The distance in our experiment exceeds all previous free-space experiments by more than one order of magnitude and exploits the limit for ground-based free-space communication; significantly longer distances can only be reached using air- or space-based platforms. The range achieved thereby demonstrates the feasibility of quantum communication in space, involving satellites or the International Space Station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Google
Pointer 28°16'20.52" N 16°38'23.52" W elev 3555 m
Streaming |||||||||| 100%
Eye alt 3.72 km

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

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Bell's theorem[1] states that certain statistical correlations predicted by quantum physics for measurements on two-particle systems cannot be understood within a realistic picture based on local properties of each individual particle—even if the two particles are separated by large distances. Einstein, Podolsky and Rosen first recognized[2] the fundamental significance of these quantum correlations (termed 'entanglement' by Schrödinger[3]) and the

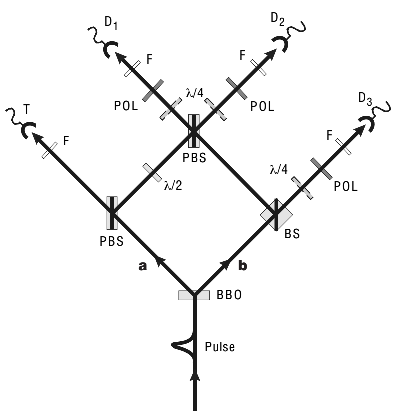

Figure 1 Experimental set-up for Greenberger–Horne–Zeilinger (GHZ) tests of quantum nonlocality. Pairs of polarization-entangled photons[28] (one photon *H* polarized and the other *V*) are generated by a short pulse of ultraviolet light (~ 200 fs, λ = 394 nm). Observation of the desired GHZ correlations requires fourfold coincidence and therefore two pairs[29]. The photon registered at T is always *H* and thus its partner in **b** must be *V*. The photon reflected at the polarizing beam-splitter (PBS) in arm **a** is always *V*, being turned into equal superposition of *V* and *H* by the λ/2 plate, and its partner in arm **b** must be *H*. Thus if all four detectors register at the same time, the two photons in D_1 and D_2 must either both have been *VV* and reflected by the last PBS or *HH* and transmitted. The photon at D_3 was therefore *H* or *V*, respectively. Both possibilities are made indistinguishable by having equal path lengths via **a** and **b** to D_1 (D_2) and by using narrow bandwidth filters (*F* ≈ 4 nm) to stretch the coherence time to about 500 fs, substantially larger than the pulse length[30]. This effectively erases the prior correlation information and, owing to indistinguishability, the three photons registered at D_1, D_2 and D_3 exhibit the desired GHZ correlations predicted by the state of equation (1), where for simplicity we assume the polarizations at D_3 to be defined at right angles relative to the others. Polarizers oriented at 45° and λ/4 plates in front of the detectors allow measurement of linear H'/V' (circular *R/L*) polarization.

ization. Then by the third term in equation (4), photon 3 will definitely be V' polarized.

By cyclic permutation, we can obtain analogous expressions for any experiment measuring circular polarization on two photons and H'/V' linear polarization on the remaining one. Thus, in every one of the three *yyx*, *yxy*, and *xyy* experiments, any individual measurement result—both for circular polarization and for linear H'/V' polarization—can be predicted with certainty for every photon given the corresponding measurement results of the other two.

Now we will analyse the implications for local realism. As these predictions are independent both of the spatial separation and of the relative time order of the three measurements, we consider them performed simultaneously in a given reference frame—say, for conceptual simplicity, in the reference frame of the source. Then, as Einstein locality implies that no information can travel faster than the speed of light, this requires any specific measurement result obtained for any photon never to depend on which specific measurements are performed simultaneously on the other two nor on their outcome. The only way then for local realism to

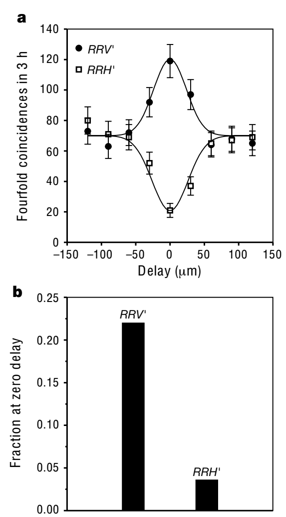

Figure 2 A typical experimental result used in the GHZ argument. This is the *yyx* experiment measuring circular polarization on photons 1 and 2 and linear polarization on the third. **a**, Fourfold coincidences between the trigger detector T, detectors D_1 and D_2 (both set to measure a right-handed polarized photon), and detector D_3 (set to measure a linearly polarized H' (lower curve) and V' (upper curve) photon as a function of the delay between photon 1 and 2 at the final polarizing beam-splitter). We could adjust the time delay between paths **a** and **b** in Fig. 1 by translating the final polarizing beam-splitter (PBS) and using additional mirrors (not shown in Fig. 1) to ensure overlap of both beams, independent of mirror displacement. At large delay, that is, outside the region of coherent superposition, the two possibilities *HHH* and *VVV* are distinguishable and no entanglement results. In agreement with this explanation, it was observed within the experimental accuracy that for large delay the eight possible outcomes in the *yyx* experiment (and also the other experiments) have the same coincidence rate, whose mean value was chosen as a normalization standard. **b**, At zero delay maximum GHZ entanglement results; the experimentally determined fractions of *RRV'* and *RRH'* triples (out of the eight possible outcomes in the *yyx* experiment) are deduced from the measurements at zero delay. The fractions were obtained by dividing the normalized fourfold coincidences of a specific outcome by the sum of all possible outcomes in each experiment—here, the *yyx* experiment.

explain the perfect correlations predicted by equation (4) is to assume that each photon carries elements of reality for both *x* and *y* measurements that determine the specific individual measurement result[5,6,8].

For photon *i* we call these elements of reality X_i with values +1(−1) for $H'(V')$ polarizations and Y_i with values +1(−1) for *R*(*L*); we thus obtain the relations[8] $Y_1Y_2X_3 = -1$, $Y_1X_2Y_3 = -1$ and $X_1Y_2Y_3 = -1$, in order to be able to reproduce the quantum predictions of equation (4) and its permutations.

We now consider a fourth experiment measuring linear H'/V' polarization on all three photons, that is, an *xxx* experiment. We investigate the possible outcomes that will be predicted by local realism based on the elements of reality introduced to explain the earlier *yyx*, *yxy* and *xyy* experiments.

Because of Einstein locality any specific measurement for *x* must be independent of whether an *x* or *y* measurement is performed on the other photon. As $Y_iY_i = +1$, we can write $X_1X_2X_3 = (X_1Y_2Y_3)(Y_1X_2Y_3)(Y_1Y_2X_3)$ and obtain $X_1X_2X_3 = -1$. Thus from a local realist point of view the only possible results for an *xxx* experiment are $V'V'V'$, $H'H'V'$, $H'V'H'$, and $V'H'H'$.

How do these predictions of local realism for an *xxx* experiment compare with those of quantum mechanics? If we express the state given in equation (1) in terms of H'/V' polarization using equation (2) we obtain:

$$|\Psi\rangle = \frac{1}{2}(|H'\rangle_1|H'\rangle_2|H'\rangle_3 + |H'\rangle_1|V'\rangle_2|V'\rangle_3 + |V'\rangle_1|H'\rangle_2|V'\rangle_3 + |V'\rangle_1|V'\rangle_2|H'\rangle_3) \qquad (5)$$

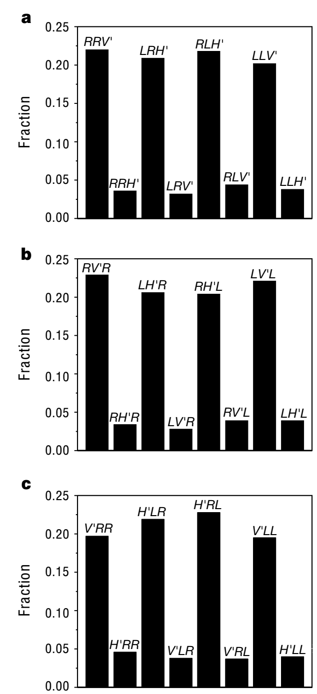

Figure 3 All outcomes observed in the *yyx*, *yxy* and *xyy* experiments, obtained as in Fig. 2. **a**, *yyx*; **b**, *yxy*; **c**, *xyy*. The experimental data show that we observe the GHZ terms predicted by quantum physics (tall bars) in a fraction of 0.85 ± 0.04 of all cases and 0.15 ± 0.02 of the spurious events (short bars) in a fraction of all cases. Within our experimental error we thus confirm the GHZ predictions for the experiments.

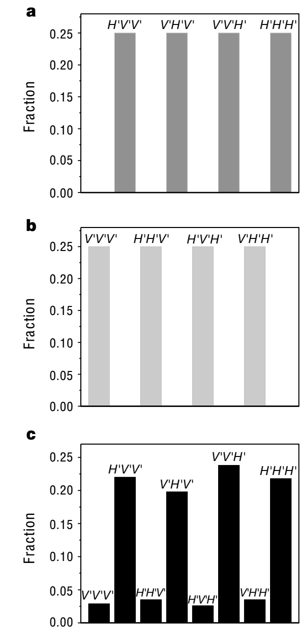

Figure 4 Predictions of quantum mechanics and of local realism, and observed results for the *xxx* experiment. **a, b,** The maximum possible conflict arises between the predictions for quantum mechanics (**a**) and local realism (**b**) because the predicted correlations are exactly opposite. **c,** The experimental results clearly confirm the quantum predictions within experimental error and are in conflict with local realism.

are shown in Fig. 2. The six remaining possible outcomes of a *yyx* experiment have also been measured in the same way and likewise in the *yxy* and *xyy* experiments. For all three experiments this results in 24 possible outcomes whose individual fractions thus obtained are shown in Fig. 3.

Adopting our first strategy, we assume that the spurious events are attributable to unavoidable experimental errors; within the experimental accuracy, we conclude that the desired correlations in these experiments confirm the quantum predictions for GHZ entanglement. Thus we compare the predictions of quantum mechanics and local realism with the results of an *xxx* experiment (Fig. 4) and we observe that, again within experimental error, the triple coincidences predicted by quantum mechanics occur and not those predicted by local realism. In this sense, we believe that we have experimentally realized the first three-particle test of local realism following the GHZ argument.

We then investigated whether local realism could reproduce the *xxx* experimental results shown in Fig. 4c, if we assume that the spurious non-GHZ events in the other three experiments (Fig. 3) actually indicate a deviation from quantum physics. To answer this we adopt our second strategy and consider the best prediction a local realistic theory could obtain using these spurious terms. How, for example, could a local realist obtain the quantum prediction $H'H'H'$? One possibility is to assume that triple events producing $H'H'H'$ would be described by a specific set of local hidden variables such that they would give events that are in agreement with quantum theory in both an *xyy* and a *yxy* experiment (for example, the results $H'LR$ and $LH'R$), but give a spurious event for a *yyx* experiment (in this case, LLH'). In this way any local realistic

Two sets of linear equality constraints, one set of linear inequalities:

Normalization; No-signalling; Non-negativity

$$
\begin{aligned}
\forall_{ab\,..} \quad & \sum_{xy\,..} p\,(xy\,..\mid ab\,..) \;=\; 1 \\
\forall_{axbb'\,..} \quad & \sum_{y\,..} p\,(xy\,..\mid ab\,..) \;=\; \sum_{y\,..} p\left(xy\,..\mid ab'\,..\right) \quad \text{etc.} \\
\forall_{ab\,..\,xy\,..} \quad & p\,(xy\,..\mid ab\,..) \;\geq\; 0
\end{aligned}
$$

Why it all works:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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