Schrödinger's cat meets Occam's razor

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Introduction

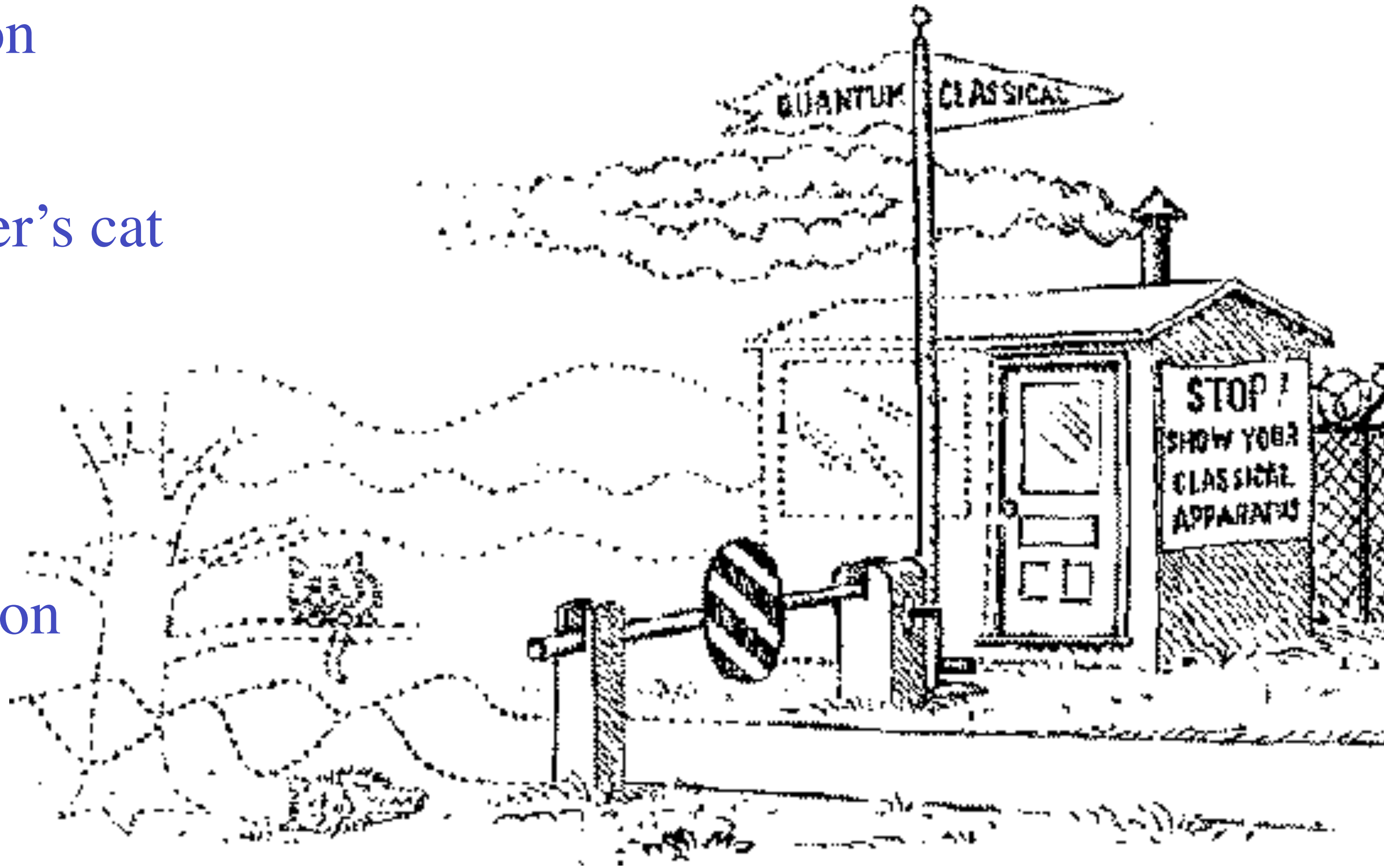
Schrödinger’s cat

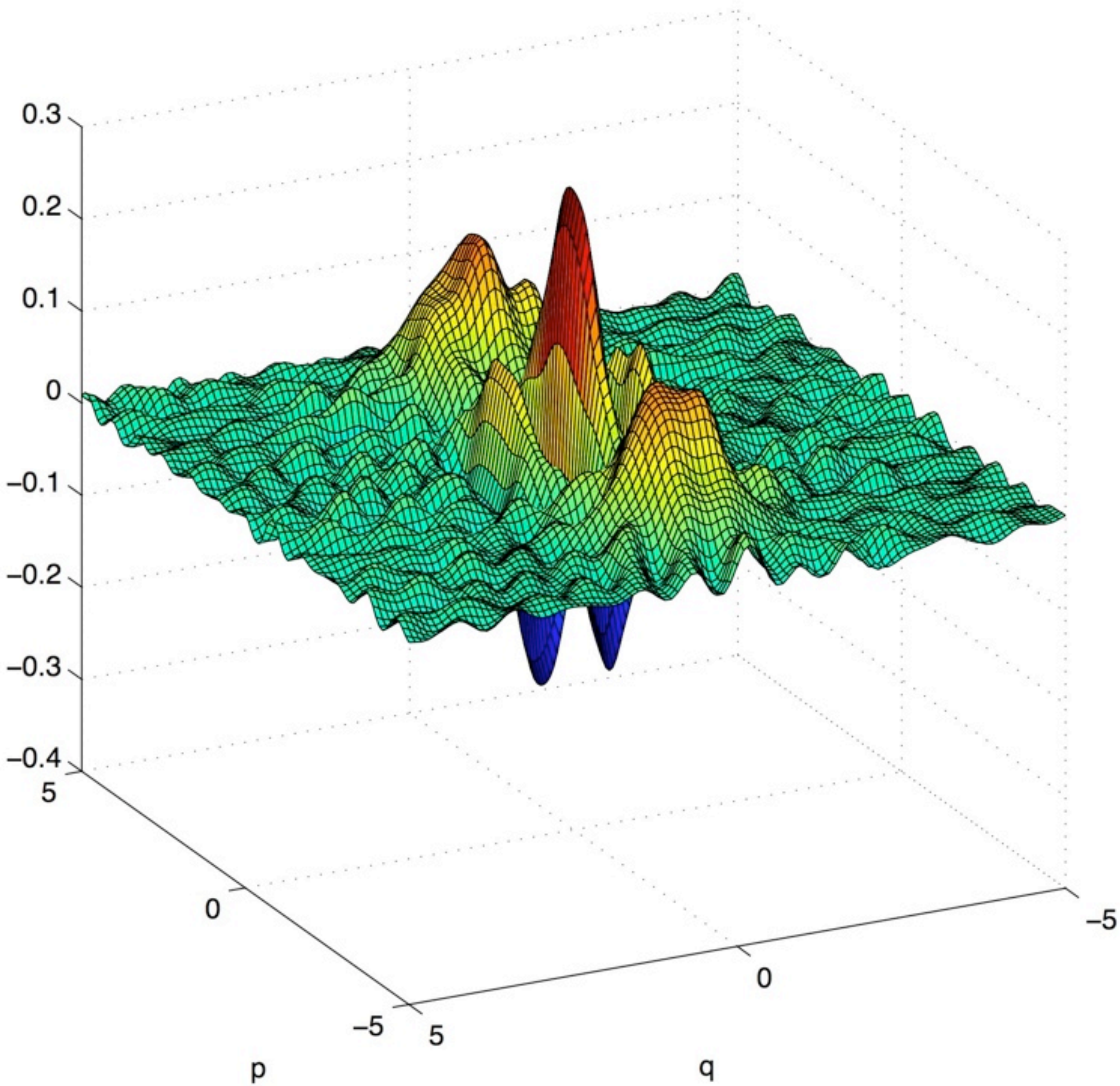
Solutions

New solution

Examples

Discussion




0.3
0.2
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5
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p
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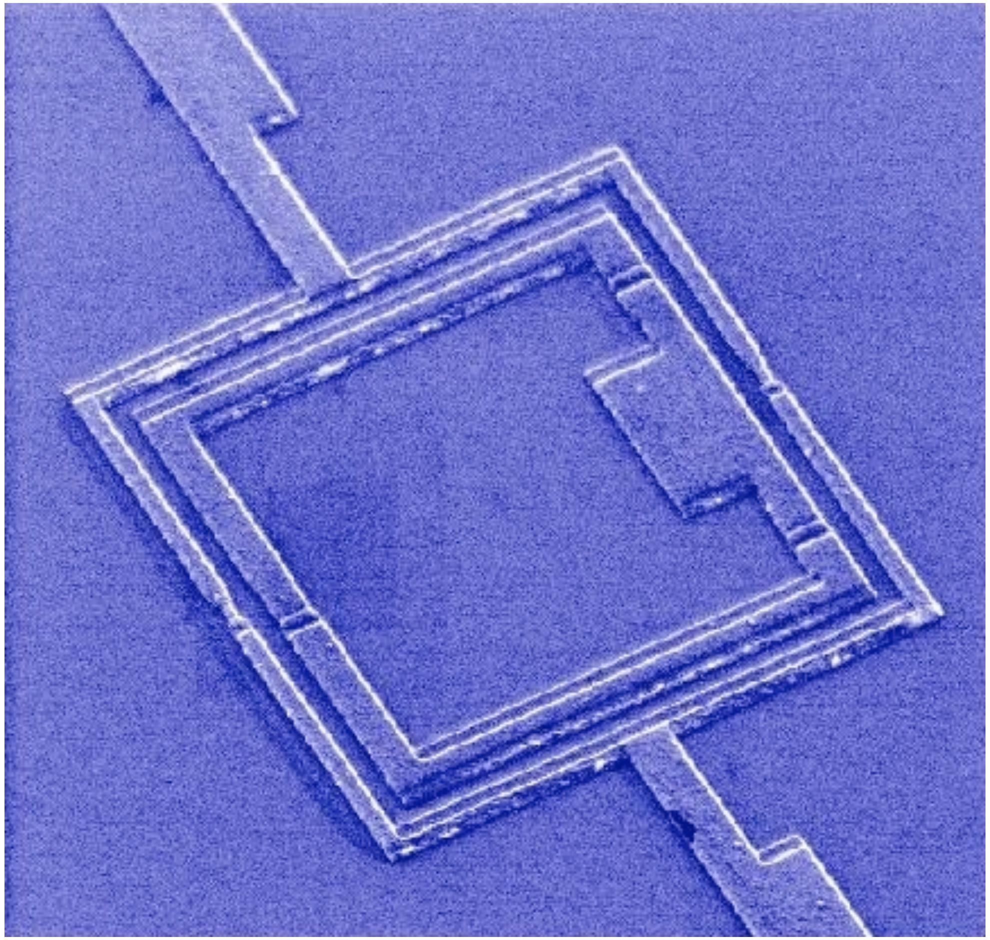
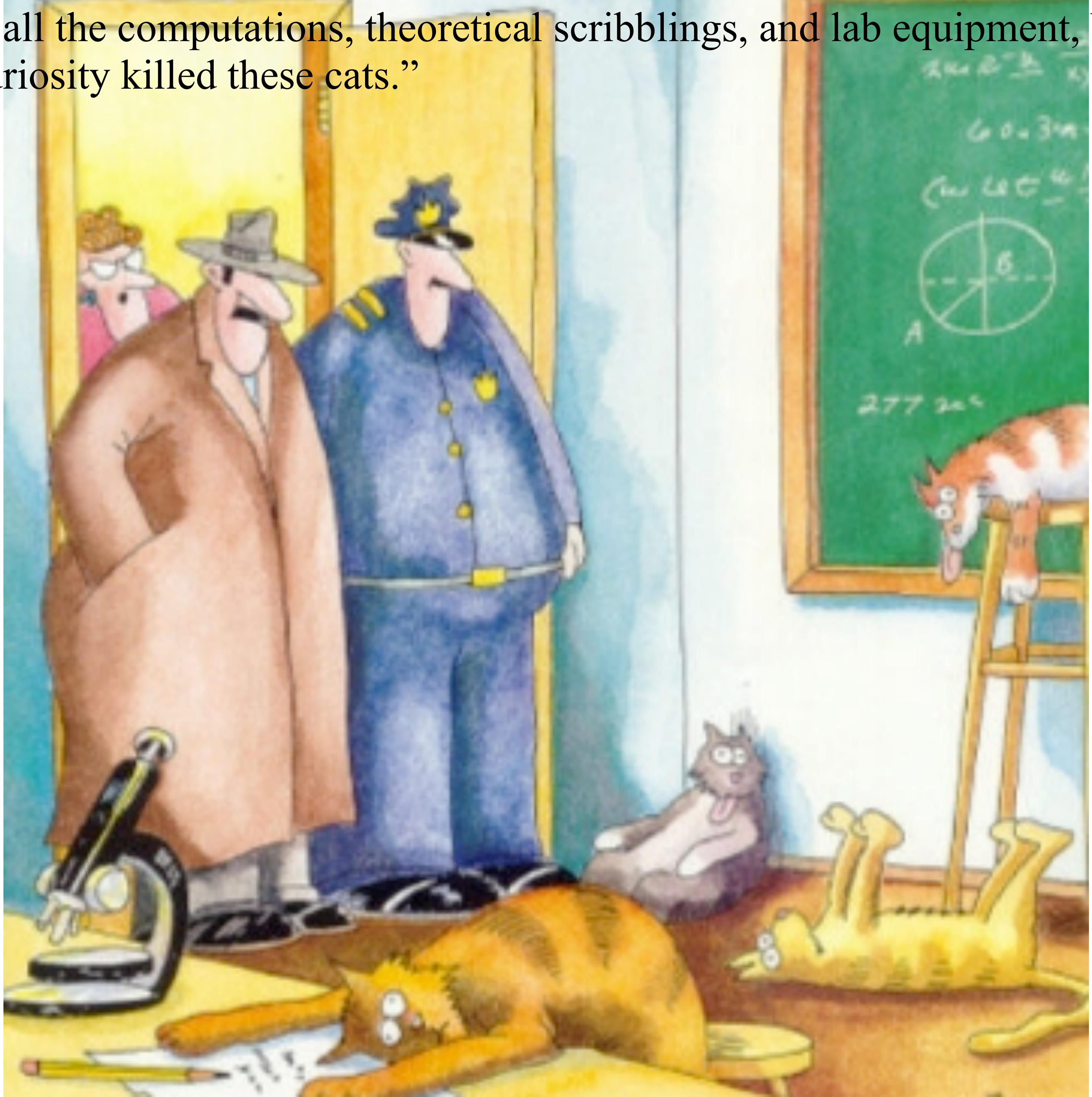
q



"Notice all the computations, theoretical scribblings, and lab equipment, Norm. Yes, curiosity killed these cats."



Baby QM

- A quantum system is described by a vector $|\psi\rangle$ in complex Hilbert space $\mathcal{H}$ such that $\|\psi\|^2 = \langle\psi|\psi\rangle = 1$
- It evolves unitarily: in discrete time, $|\psi\rangle \mapsto U|\psi\rangle$ where $UU^* = U^*U = I$, hence $U^{-1} = U^*$
- Thus at time $n \in \mathbb{Z}$, the system is in state $U^n|\psi\rangle$
- A von Neumann measurement of the quantum system corresponds to a decomposition of $\mathcal{H}$ into orthogonal closed subspaces labelled by the measurement outcomes $x \in \mathcal{X}$
- The outcome is x and the state jumps to $\Pi_x|\psi\rangle/\sqrt{p(x)}$ with probability $p(x) = \|\Pi_x|\psi\rangle\|^2$
- $\Pi_x = \Pi_x^2 = \Pi_x^* =$ orthogonal projection into subspace "x"



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Toddler QM, “Schrödinger picture”

- Define $\rho = |\psi\rangle\langle\psi|$, suppose $x \in \mathbb{R}$ and define $X = \sum_x x \Pi_x$; define $f(X) = \sum_x f(x)\Pi_x$
- The evolution of the state is $\rho \mapsto U\rho U^*$
- Measurement of X yields random x with probability trace $\rho\Pi_x$; expectations given by $\langle f(x)\rangle =$ trace $\rho f(X)$
- The state jumps to $\Pi_x\rho\Pi_x/p(x)$, under which the probability law of measurement of X is δ_x
- Extends to probabilistic mixtures: $\rho = \sum_x p_x|\psi_x\rangle\langle\psi_x|$

Toddler QM, “Heisenberg picture”

- Define $\rho = |\psi\rangle\langle\psi|$, suppose $x \in \mathbb{R}$ and define $X = \sum_x x \Pi_x$; define $f(X) = \sum_x f(x)\Pi_x$
- The evolution of an *observable* is $X \mapsto U^* X U$
- Measurement of X yields random x with $\langle f(x) \rangle = \text{trace}\, \rho f(X)$
- The *state* jumps to $\Pi_x \rho \Pi_x / p(x)$, under which the probability law of measurement of X is δ_x
- $\ldots$ all this because $\text{trace} U \rho U^* f(X) = \text{trace} \rho f(U^* X U)$

Kindergarten QM

- A state ρ is a nonnegative operator with trace 1
- A quantum operation on a state producing an outcome x is described by a collection of operators A_{xy} such that

$$\sum_{x,y} A_{xy}^* A_{xy} = I$$

- The outcome is x with probability

$$p(x) = \text{trace}\ \rho \sum_{y} A_{xy}^* A_{xy}$$

and the state jumps to

$$\sum_{y} A_{xy} \rho A_{xy}^* / p(x)$$

Kraus, Naimark, Stinespring

Theory : the following are equivalent

- operator-sum representation
- completely positive norm preserving maps (physical properties implied by mixture interpretation of $\rho = \sum p_i \rho_i$)
- bringing separate systems together into a composite system, unitary evolution, von Neumann measurement, discarding components of composite systems

More precisely, any quantum operation can be realised as follows:

- System of interest A meets ancillary system B
  $\rho = \rho^A \mapsto \rho^A \otimes \rho_0^B = \rho^{AB}$
- Unitary evolution $\rho^{AB} \mapsto U^{AB} \rho^{AB} U^{AB*}$
- Measure B von Neumann-wise
- Discard B: define ρ^A by

$$\text{trace}(\rho^A X^A) = \text{trace}(\rho^{AB}\ X^A \otimes I^B) \qquad \forall\ X^A$$

The Church of the Larger Hilbert Space

Suppose $|\psi\rangle^{AB}$ is a pure state on AB.
Schmidt: we can choose o.n.b.'s such that

$$|\psi\rangle^{AB} = \sum \lambda_i \; |i\rangle^A \otimes |i\rangle^B, \qquad \lambda_i \in \mathbb{R}_+$$

If we discard B the state of A is

$$\rho^A = \mathrm{trace}_B(\rho^{AB}) = \sum \lambda_i^2 \; |i\rangle^A \langle i|^A$$

So we can forget about probability altogether ...(?)

Schrödinger's cat

- Atom in state $|0\rangle$ does nothing
- Atom in state $|1\rangle$ emits a particle, decaying to state $|0\rangle$
- Particle is registered by detector
  which releases poison killing the cat
- Atom in state $\alpha|0\rangle + \beta|1\rangle$: cat killed with probability $|\beta|^2$

But atom, detector, poison, cat are one quantum system

- $|0\rangle \otimes |\text{live cat}$ evolves to $|0\rangle \otimes |\text{live cat}\rangle$
- $|1\rangle \otimes |\text{live cat}$ evolves to $|0\rangle \otimes |\text{dead cat}\rangle$
- Hence $\left(\alpha\,|0\rangle + \beta\,|1\rangle\right) \otimes |\text{live cat}\rangle$
  evolves to
  $|0\rangle \otimes \left(\alpha\,|\text{live cat}\rangle + \beta\,|\text{dead cat}\rangle\right)$

But such cats have never been seen …

A myriad solutions

- QM is wrong; Schrödinger's equation should be modified
  Add stochastic term, due to gravity (C.S.L., Penrose)
- There is no problem: predictions have been tested in the lab.
- There is no problem: as Bohr said, we must always assume "Heisenberg cut" between quantum and classical level
- Many worlds
- Bohm: nonlocal, deterministic hidden layer
- Church of the larger Hilbert space
- Physicist's solution:
  - Model system A, (pointer of) device B, environment C; after interaction, when we discard C,
    ρ^{AB} is diagonal in basis of pointer (Zurek)
  - Model system A, (pointer of) device B, environment C; environment is initially in a mixed state; after interaction
    ρ^{ABC} is diagonal in basis of pointer (Nieuwenhuizen)

NB the environment can be an *in*vironment

Objections

- The physicists' solutions show that for toy models,
  in the limit of many particles, long time …
  classical behaviour "seems to emerge"
  However the limiting situation is "outside the model"
- Many solutions involve a "preferred basis" thus already build in what they are supposed to predict
- Many solutions are only solutions because they restrict the domain of discourse
- Many solutions are merely word games
  "We need a more general notion of"
  probability, logic, reality ...
  Many worlds = many words?

New solution

- Hepp: algebraic approach, *emergence* of superselection rules
- Belavkin, Landsman

Philosophy: look for niche for “life as we know it”
within purely quantum universe.
Characterized by causality (time ordering, spatial separation)

- Given: a unitary U and a state ρ on Hilbert space $\mathcal{H}$
- $\mathcal{B}(\mathcal{H})$: *all* bounded operators (all “observables”)
- $\mathcal{C} \subseteq \mathcal{B}(\mathcal{H})$, a set of *beables*:
  a *commuting unital von Neumann algebra*
- $U\mathcal{C}U^* \subseteq \mathcal{C}$: the causality principle; allows the *beables* to be *viable* (backwards Heisenberg picture)

In case you had forgotten:

- A unital von Neumann algebra: subset of $\mathcal{B}(\mathcal{H})$ algebraically closed for addition, composition, $*$, scalar multiplication; containing the identity; closed under topology of weak convergence: $X_n \to X$ iff trace$(\rho X_n) \to$ trace(ρX) for all ρ
- Every *commuting* von Neumann algebra is isomorphic to some $L^\infty(\Omega, \mathcal{F}, P)$

Define *predictables* $\mathcal{A} = \mathcal{C}'$ (commutant of $\mathcal{C}$). Then

- $\mathcal{C} = \mathcal{A}'$
- $U\mathcal{C}U^* \subseteq \mathcal{C}$ iff $U^*\mathcal{A}U \subseteq \mathcal{A}$

Thus we have

$$\mathcal{C} \subseteq \mathcal{A} \subseteq \mathcal{B}(\mathcal{H})$$

Beables (viables) $\subseteq$ predictables $\subseteq$ observables

Past beables are beable; the beables are viable!

Future predictables are predictable, too

Any state ρ, restricted to $\mathcal{A}$, can be *uniquely* decomposed as *probabilistic mixture* over $x \in \Omega$ of quantum states ρ_x on $\mathcal{A}$ (concentrating on x, when restricted to $\mathcal{C}$).

Conditional probabilities in QM *only* exist in this case!!

Restricted to $\mathcal{C}$,
the *backwards* Schrödinger evolution is deterministic,
the forwards Schrödinger evolution is stochastic.

(the conditional state on $\mathcal{A}$, given x in Ω)

The state ρ_x encapsulates the probability distribution of the future evolution of x

(in Ω)

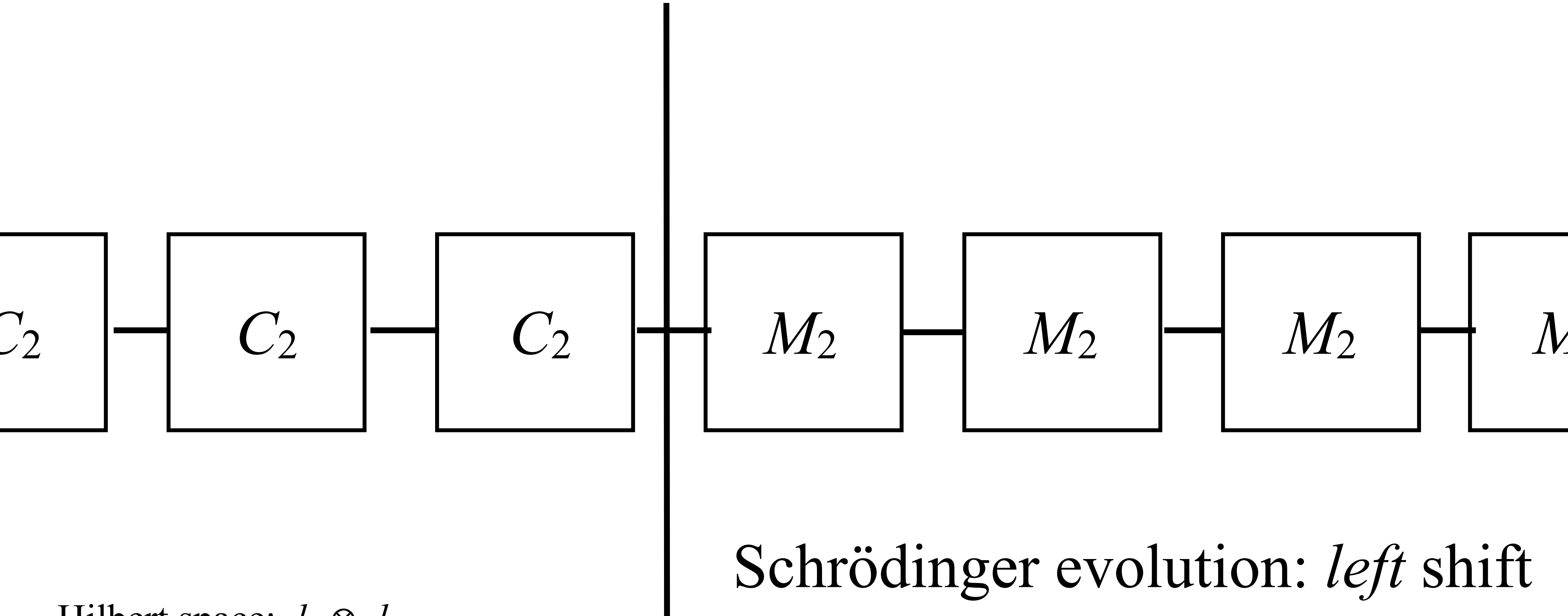
The physics of a viable world is *not* time-reversible

Note: $\mathcal{H}$ *must* be infinite dimensional for non-trivial examples

Examples

Everything in quantum information
Everything in quantum optics (continuous time!)
The C.S.L. model

Example 1



Schrödinger evolution: *left* shift

Hilbert space: $l_2 \otimes l_2$
(closed subspace of two-sided tensor product of countable no. of copies of $\mathbb{C}^2$)

o.n.b.'s labelled by pair of nonneg. integers written in binary; reversed binary
e.g. $| 1\ 0 ; 0\ 0\ 1 \rangle = | 2 ; 4 \rangle$

Example 1 (cont.)

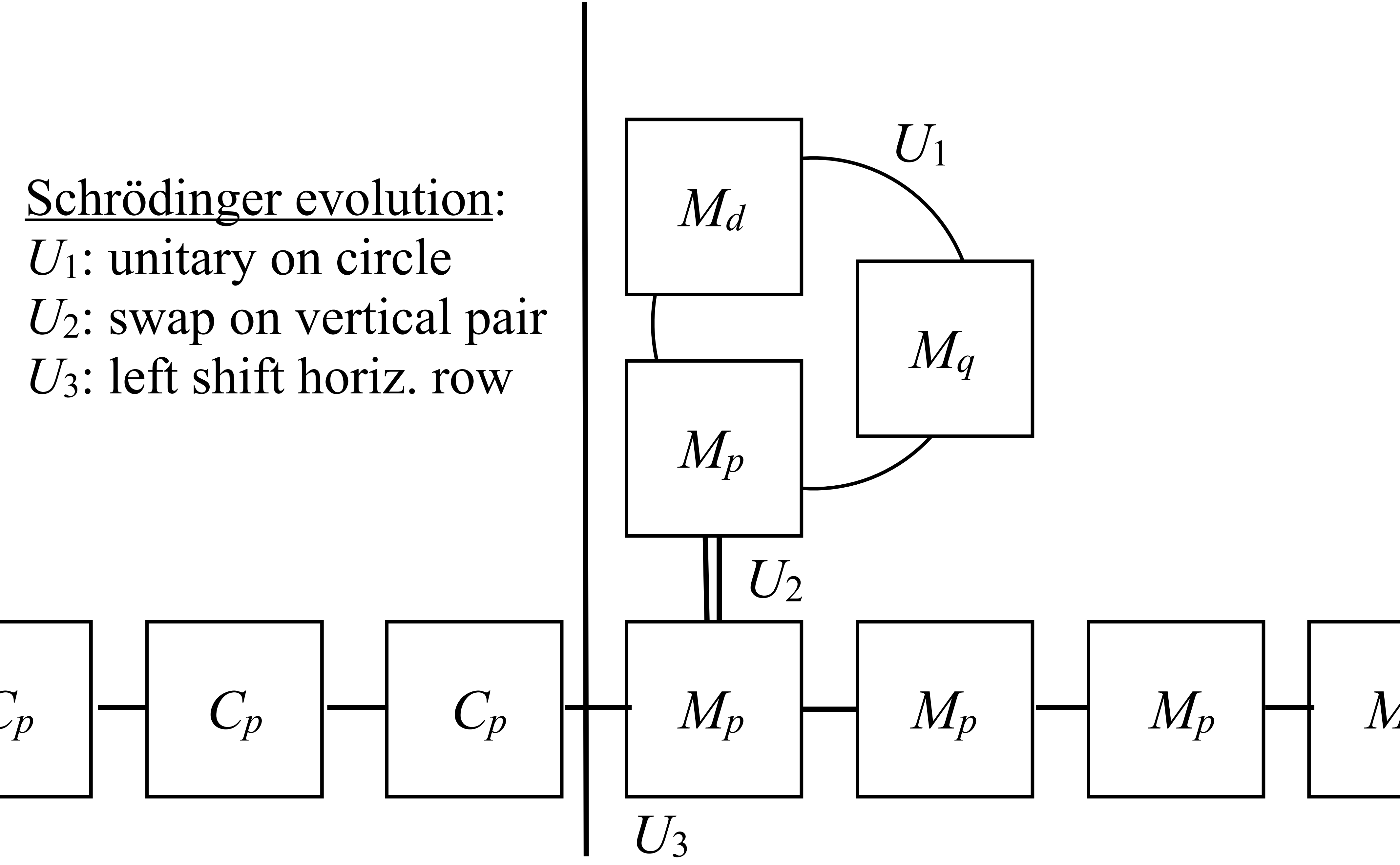
- Pure state $| n ; c_0, c_1, c_2, \ldots \rangle$, $\sum_m |c_m|^2 = 1$
- Schrödinger evolution: new (unnorm.) state is
  $| 2n + 0 ; c_0, c_2, \ldots \rangle$ or $| 2n + 1 ; c_1, c_3, \ldots \rangle$

  with probabilities

  $\sum_{m \text{ even}} |c_m|^2$ and $\sum_{m \text{ odd}} |c_m|^2$ respectively

Example 2

<u>Schrödinger evolution</u>:
U_1: unitary on circle
U_2: swap on vertical pair
U_3: left shift horiz. row



Theorem

- This picture is generic – see my cat paper, first section, where I analyse the case
  $\mathcal{H} = l_2 \otimes l_2$
  $\mathcal{A}$: everything which commutes with number operator on first component,
  $X \mid n\,;m\rangle = n \mid n\,;m\rangle$
  $\mathcal{C}$: all functions of X

Discussion

- Given $\mathcal{H}$ and U there typically exist *many* mutually incompatible, non-trivial, $\mathcal{C}$
- Our universe allows many incompatible "real worlds"!
- We live in one of them. Our notions of causality are derived from our notions of space (separation) and time (direction) (or vice-versa)
- Space-separation – commutativity – locality
- Given partial information about $\mathcal{C}$, the *causality* assumption could fix other properties of it
- Thus the classical world $\mathcal{C}$ can indeed emerge from causality/locality and from the laws of quantum physics

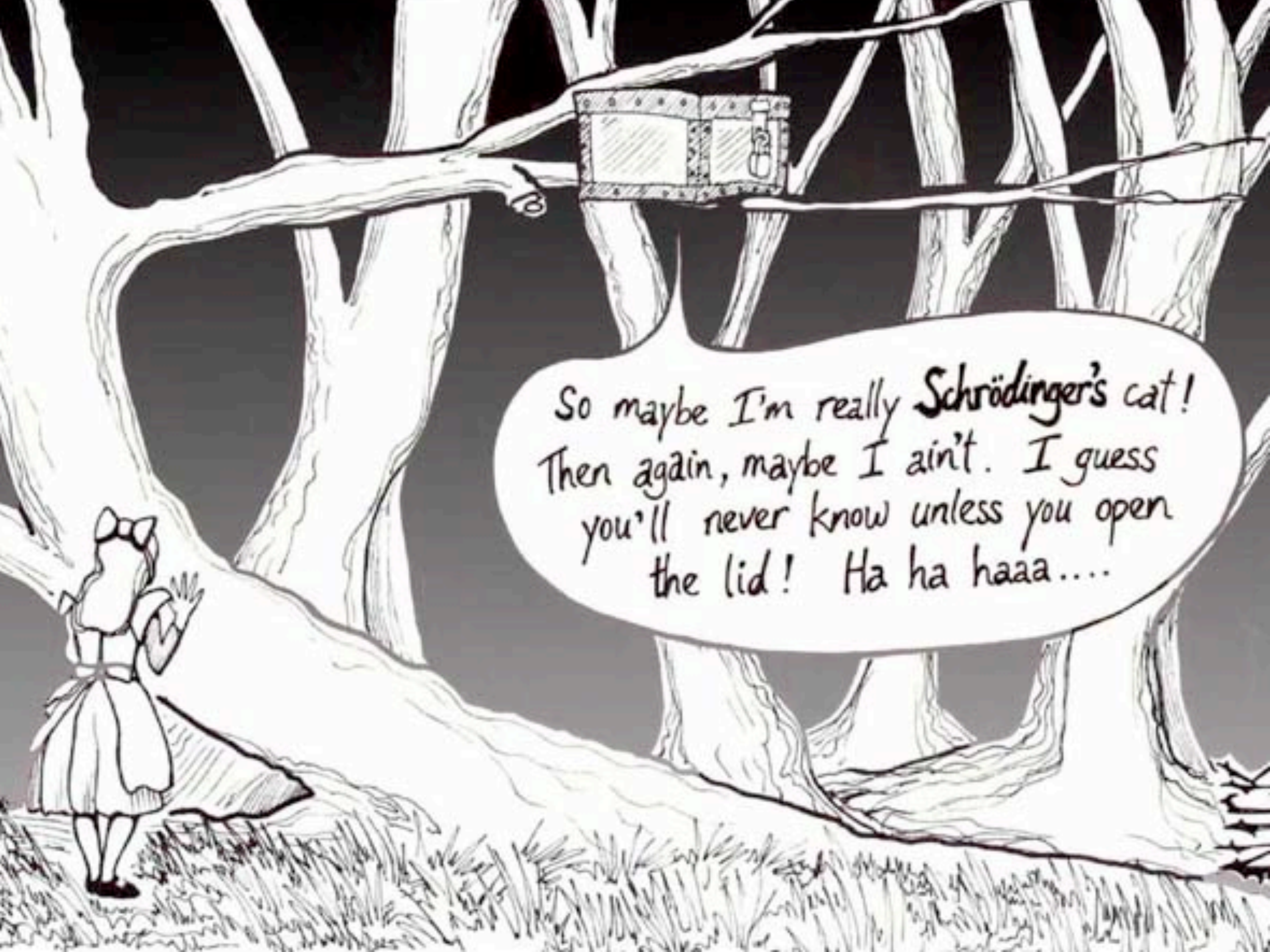
- Physics must show how time and space are emergent properties
- Space-time exists by virtue of gravity which exists by virtue of matter having a location in space-time
- Particles have trajectories in the past, but are waves in the future
  Past space-time is frozen, future space-time is still to be born
- Quantum measurement = continual stochastic birth of space-time

- A-realism or B-realism? – the distinction is purely academic!
  - A realism: only relative (inter-subjective) reality exists
    Landsman (an A-realist): B-realists are hallucinating
  - B realism: there exists a unique objective reality
    RDG (a B-realist): A realists are hallucinating
- Is the aim of physics merely to predict, or is it to understand?
  What is the role of mathematics?
  Provides tools to calculate, or models to give understanding?

- N.P. Landsman (1995)
  Observation and superselection in quantum mechanics
  Studies in the History and Philosophy of Modern Physics **26**
  1355–2198

- V.P. Belavkin (2007)
  Eventum Mechanics of Quantum Trajectories: Continual Measurements, Quantum Predictions and Feedback Control
  Submitted to *Reviews on Mathematical Physics*
  `arXiv.org: math-ph/0702083`

- R.D. Gill (2009)
  Putting Schrödinger's cat to rest
  Preliminary draft


So maybe I'm really Schrödinger's cat! Then again, maybe I ain't. I guess you'll never know unless you open the lid! Ha ha haaa....