StAN Exercise Sheet 2

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You are welcome to solve the following questions by simulation experiments (using `R`) rather than theoretical calculations.

1 Moment estimators

1.1 Uniform distribution

Suppose $X_1, \ldots, X_n$ is a random sample of size n from the uniform distribution on the interval (α, β) where $-\infty < \alpha < \beta < \infty$ are unknown parameters.

What are the moment estimators of α and β?

What can you say about mean, variance, mean square error, consistency of these estimators?

1.2 Pareto distribution

Suppose $X_1, \ldots, X_n$ is a random sample of size n from the Pareto distribution (power law distribution) with shape parameter α and location parameter β: each observation has probability density function $f(x|\alpha, \beta) = \alpha \beta^{\alpha} x^{-\alpha-1}$ for $x \geq \beta$, $f(x|\alpha, \beta) = 0$ for $x < \beta$.

Verify that $f(x|\alpha, \beta)$ is indeed a probability density function for any $\alpha > 0$ and $\beta > 0$.

What are mean and variance of this probability distribution?

What are the method-of-moments estimators of α and β?

What can you say about mean, variance, mean square error of these estimators? Are they consistent?

1.3 Gamma distribution

Suppose $X_1, \ldots, X_n$ is a random sample of size n from the gamma distribution with shape parameters $\nu > 0$ and inverse scale paramour $\lambda > 0$: each observation has probability density function $f(x|\lambda, \nu) = \lambda^\nu x^{\nu-1} \exp(-\lambda x)/\Gamma(\nu)$ for $x > 0$, $f(x|\lambda, \nu) = 0$ for $x < 0$.

What are method-of-moment estimators of λ and ν?

What can you say about mean, variance, mean square error of these estimators? Are they consistent?

2 Maximum likelihood estimators

2.1 Uniform distribution

Analyse the maximum likelihood estimators of α and β and compare with the method-of-moment estimators.

2.2 Pareto distribution

Analyse the maximum likelihood estimators of α and β and compare with the method-of-moment estimators.

2.3 Gamma distribution

Analyse the maximum likelihood estimators of λ and ν and compare with the method-of-moment estimators.